6.4 An Application to the Calendar

Note Title 8/11/2005

1. Find The number n of leap years s.t. $1600 < n < Y$, for

(a) $Y = 1825$ $(c = 18, y = 25)$

$$L = 24c + \left[\frac{c}{4}\right] + \left[\frac{y}{4}\right] - 388,$$

$$= 24(18) + \left[\frac{18}{4}\right] + \left[\frac{25}{4}\right] - 388$$

$$= 432 + 4 + 6 - 388 = \underline{\underline{54}}$$

(b) $Y = 1950$ $(c = 19, y = 50)$

$$L = 24c + \left[\frac{c}{4}\right] + \left[\frac{y}{4}\right] - 388$$

$$= 24(19) + \left[\frac{19}{4}\right] + \left[\frac{50}{4}\right] - 388$$

$$= 456 + 4 + 12 - 388 = \underline{84}$$

(c) $Y = 2075$ $(c = 20, y = 75)$

$$L = 24c + \left[\frac{c}{4}\right] + \left[\frac{y}{4}\right] - 388$$

$$= 24(20) + \left[\frac{20}{4}\right] + \left[\frac{75}{4}\right] - 388$$

$$= 480 + 5 + 18 - 388 = \underline{\underline{115}}$$

2. Determine the day of the week for which you were born.

Jan. 12, 1952. $c = 19$, $y = 51$, $m = 11$, $d = 12$

$$W \equiv d + [(2.6)m - 0.2] - 2c + y + [\tfrac{c}{4}] + [\tfrac{y}{4}] \pmod 7$$

$$= 12 + [2.6(11) - 0.2] - 2(19) + 51 + [\tfrac{19}{4}] + [\tfrac{51}{4}] \pmod 7$$

$$= 12 + 28 - 38 + 51 + 4 + 12 \pmod 7$$

$$= 69 = 9 \cdot 7 + 6 \equiv 6 \pmod 7 \quad 6 \Rightarrow \underline{\text{Sat}}$$

3. Find the day of the week for the important dates:

(a) November 19, 1863 (Lincoln's Gettysburg Address).

$$w \equiv d + [2.6m - 0.2] - 2c + y + [\tfrac{c}{4}] + [\tfrac{y}{4}] \pmod 7$$

$$= 19 + [2.6(9) - 0.2] - 2(18) + 63 + [\tfrac{18}{4}] + [\tfrac{63}{4}]$$

$$= 19 + 23 - 36 + 63 + 4 + 15 = 88 = 7 \cdot 12 + 4$$

$$\equiv 4 \pmod 7 \Rightarrow \underline{\text{Thu}}$$

(b) April 18, 1906 (S.F. earthquake)

$$w \equiv d + [2.6m - 0.2] - 2c + y + \left[\frac{c}{4}\right] + \left[\frac{y}{4}\right] \pmod 7$$

$$= 18 + [2.6(2) - 0.2] - 2(19) + 6 + \left[\frac{19}{4}\right] + \left[\frac{6}{4}\right]$$

$$= 18 + 5 - 38 + 6 + 4 + 1 = -4 \equiv 3 \pmod 7 \Rightarrow \underline{\text{Wed}}$$

(c) Nov. 11, 1918 (Great War ends)

$$w \equiv 11 + [2.6(9) - 0.2] - 2(19) + 18 + \left[\frac{19}{4}\right] + \left[\frac{18}{4}\right]$$

$$= 11 + 23 - 38 + 18 + 4 + 4 = 22 \equiv 1 \pmod 7 \Rightarrow \underline{\text{Mon}}$$

(d) Oct. 24, 1929 (N.Y. stock market crash)

$$w \equiv 24 + [2.6(8) - 0.2] - 2(19) + 29 + \left[\frac{19}{4}\right] + \left[\frac{29}{4}\right]$$

$$= 24 + 20 - 38 + 29 + 4 + 7 = 46 \equiv 4 \pmod 7$$

$$\Rightarrow \underline{\text{Thu}}$$

(e) June 6, 1944 (D-Day, Allies land in Normandy)

$$w \equiv 6 + [2.6(4) - 0.2] - 2(19) + 44 + \left[\frac{19}{4}\right] + \left[\frac{44}{4}\right]$$

$= 6 + 10 - 38 + 44 + 4 + 11 = 37 \equiv 2 \pmod 7$

$\Rightarrow$ Tue

(f) Feb. 15, 1898 (Battleship Maine blown up).

$W \equiv 15 + [2.6(12) - 0.2] - 2(18) + 97 + [\frac{18}{4}] + [\frac{97}{4}]$

$= 15 + 31 - 36 + 97 + 4 + 24 = 135 = 7 \cdot 19 + 2$

$\equiv 2 \pmod 7 \Rightarrow$ Tue

4. Show that days with the identical calendar date in the years 1999 and 1915 fell on the same day of the week.

Pf: Let W_1 = weekday for any day in 1915
W_2 = weekday for any day in 1999

The months and days will be the same.

$\therefore$ For 1915:

$W_1 \equiv d + [2.6m - 0.2] - 2(19) + 15 + [\frac{19}{4}] + [\frac{15}{4}]$

$\equiv d + [2.6m - 0.2] - 38 + 15 + 4 + 3$

$\equiv d + [2.6m - 0.2] - 16 \pmod 7$

$$\equiv d + [2.6m - 0.2] + 5 \pmod 7$$

For 1999:

$$w_2 \equiv d + [2.6m - 0.2] - 2(19) + 99 + \left[\frac{19}{4}\right] + \left[\frac{99}{4}\right]$$

$$\equiv d + [2.6m - 0.2] - 38 + 99 + 4 + 24$$

$$\equiv d + [2.6m - 0.2] + 89 \pmod 7$$

$$\equiv d + [2.6m - 0.2] + 5 \pmod 7$$

$\therefore w_1 - w_2 \equiv 0 \pmod 7$, or $w_1 \equiv w_2 \pmod 7$

5. For the year 2010, determine the following:

(a) The calendar dates on which Mondays will occur in March.

Monday $\Rightarrow 1$, so

$$1 \equiv d + [2.6(1) - 0.2] - 2(20) + 10 + \left[\frac{20}{4}\right] + \left[\frac{10}{4}\right]$$

$$= d + 2 - 40 + 10 + 5 + 2 = d - 21 \pmod 7$$

$\therefore 22 \equiv d \pmod 7$, or $1 \equiv d \pmod 7$

$\therefore$ March 1, 8, 15, 22, and 29 will all be Mondays in 2010

(b) The months in which the 13th will fall on Fri.

Friday $\Rightarrow 5$, so

$5 \equiv 13 + [2.6m - 0.2] - 2(20) + 10 + [\frac{20}{4}] + [\frac{10}{4}]$

$= [2.6m - 0.2] + 13 - 40 + 10 + 5 + 2$

$= [2.6m - 0.2] - 10 \pmod 7$

$\therefore 15 \equiv 1 \equiv [2.6m - 0.2] \pmod 7$

$m = 1, 2, \ldots, 9, 10$, $[2.6m - 0.2]$ becomes

$2, 5, 7, 10, 12, 15, 18, 20, 23, 25$

mod 7, These values are $2, 7, 0, 3, 5, 1, 4, 6, 2, 4$

$\therefore$ When $m = 6$, $[2.6m - 0.2] \equiv 1 \pmod 7$

$m = 6 \Rightarrow$ August, so August 2010 will contain a Friday 13.

Now must check Jan & Feb 2010,
which are in year 2009 for the formula.

$$5 \equiv 13 + [2.6m - 0.2] - 2(20) + 9 + \left[\frac{20}{4}\right] + \left[\frac{9}{4}\right]$$

$$= [2.6m - 0.2] - 40 + 9 + 5 + 2$$

$$= [2.6m - 0.2] - 24 \pmod 7$$

$$\therefore 29 \equiv 1 \equiv [2.6m - 0.2] \pmod 7$$

For $m = 11, 12$, $[2.6m - 0.2] = 28, 31$

And $28 \not\equiv 1$ and $31 \not\equiv 1 \pmod 7$, so
Jan, Feb. contain no Fri. 13 for 2010.

$\therefore$ For 2010, August is only month with Fri. 13

6. Find the years in the decade 2000 to 2009
when Nov. 29 is on a Sunday.

Nov. $\Rightarrow m = 9$, Sunday $\Rightarrow w = 0$

$$\therefore 0 \equiv 29 + [2.6(9) - 0.2] - 2(20) + Y + \left[\frac{20}{4}\right] + \left[\frac{Y}{4}\right]$$

$= 29 + 23 - 40 + y + 5 + [\frac{y}{4}]$

$= 17 + y + [\frac{y}{4}]$

$\therefore 4 \equiv y + [\frac{y}{4}] \pmod 7, \quad 0 \leq y \leq 9$

For $y = 0, 1, 2, 3, 4, 5, 6, 7, 8$ $\quad 4 \not\equiv y + [\frac{y}{4}] \pmod 7$

For $y = 9$, $\quad y + [\frac{y}{4}] = 11 \equiv 4 \pmod 7$

$\therefore$ Nov. 29 is a Sunday only for <u>2009</u>.