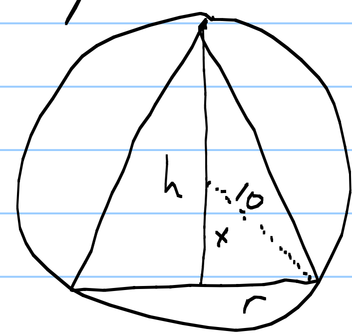
17. **17.** A right circular cone is inscribed in a sphere of radius 10 units. Express the volume, V, of the cone as the value of a function whose independent variable is the altitude, h, of the cone.

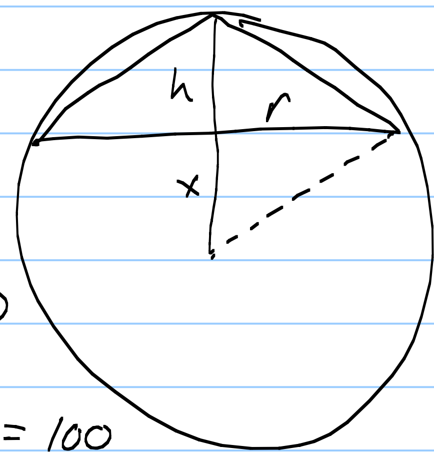
For a right circular cone, its height will be on a diameter of the sphere.



For $h \geq 10$, $x + 10 = h$

For $h < 10$, $10 - x = h$

For both, $r^2 + x^2 = 10^2$



$\therefore r^2 = 100 - x^2$

$\therefore h \geq 10 : x^2 = h^2 - 20h + 100$

$\quad h < 10 : x^2 = h^2 - 20h + 100$

$\therefore r^2 + x^2 = r^2 + h^2 - 20h + 100 = 100$

$\therefore r^2 = 20h - h^2$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(20h^2 - h^3)$

4. In Exercises 3 and 4, $f(x) = 1/(3x - 1)$ and $g(x) = \sqrt{x}$.

4. Find $(f \circ g)(x)$ and give its domain.

$$f(g(x)) = \frac{1}{3\sqrt{x} - 1}$$

$D_g = x \geq 0 \qquad D_f :$ all x, $x \neq \frac{1}{3}$

$\therefore D_{f \circ g}$ $x \geq 0$ s.t. $\sqrt{x} \neq \frac{1}{3}$

i.e. $x \neq \frac{1}{9}$

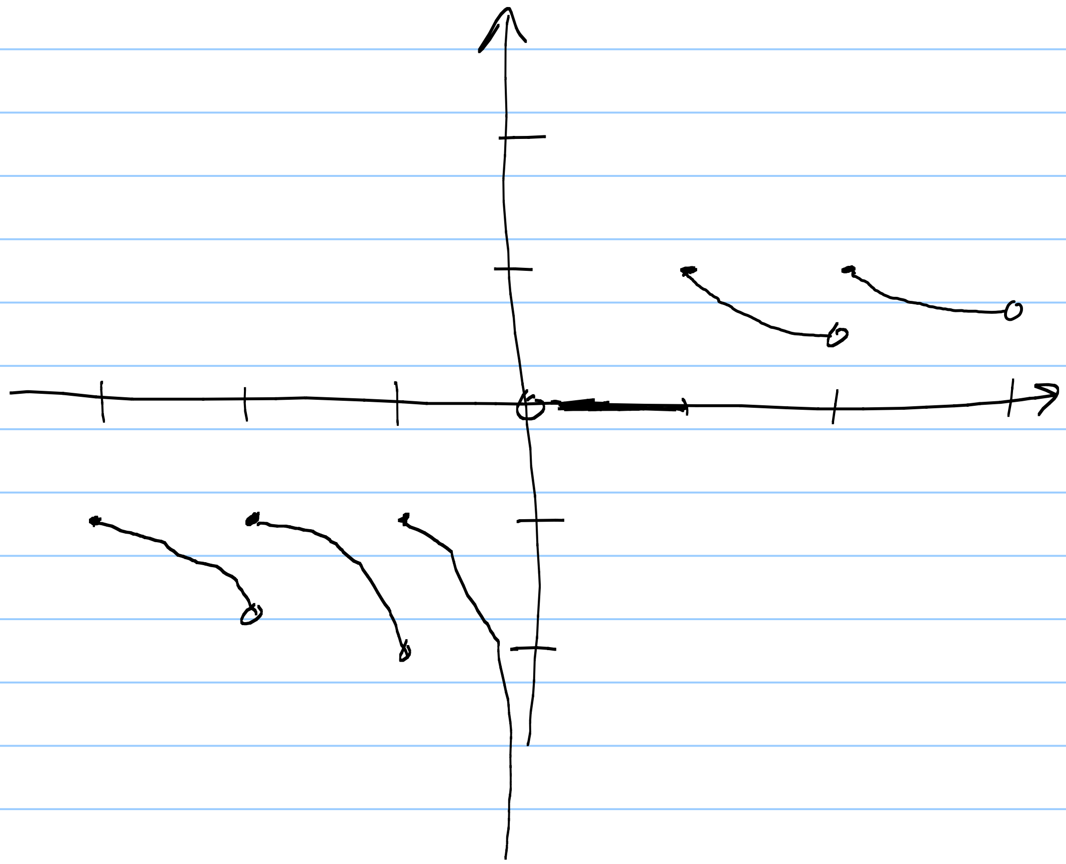
$\therefore D_{f \circ g} = \{x : x \neq \frac{1}{9}$ and $x \geq 0\}$.

3.3 Special Functions

Note Title 8/4/2014

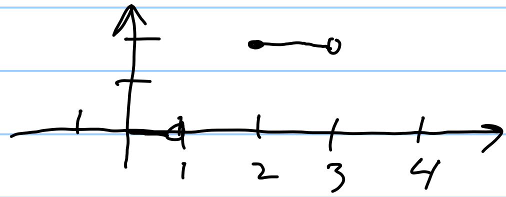
13. In Exercises 9–13, sketch the graph of the function whose rule of correspondence is given, and state its domain and range.

13. $h(x) = \dfrac{[x]}{|x|}$.



16. **16.** $F(t) = \begin{cases} [t] & \text{when } [t] \text{ is even,} \\ 2t - [t+1] & \text{when } [t] \text{ is odd.} \end{cases}$

$F(t) = [t]$, $[t]$ even :

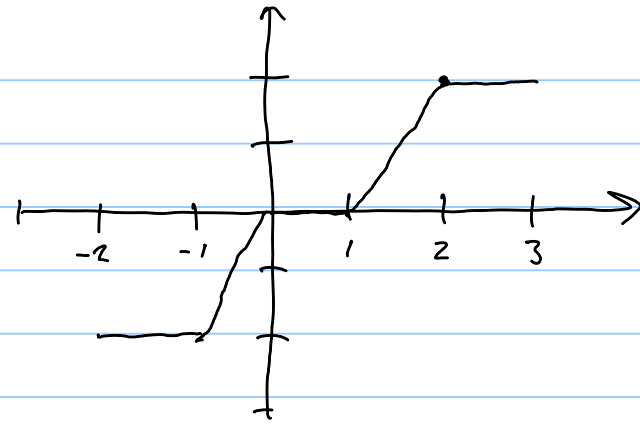


for $1 \le t < 2$, $[t]$ is 1

$[t+1]$ is 2

$\therefore F(t) = 2t - 2$. $2t - 2 = 0$ at $t = 1$, 2 at $t = 2$.

$\therefore f(x)$:

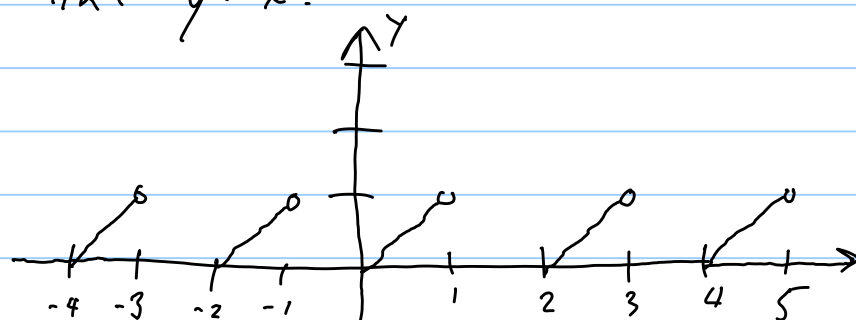


17.

17. $f(x) = \begin{cases} |x - [x]| & \text{when } [x] \text{ is even,} \\ |x - [x+1]| & \text{when } [x] \text{ is odd.} \end{cases}$

When $[x]$ is even $f(x) = y$, $0 \le y < 1$
and acts like $y = x$.

$\therefore$



$|-1.9 - [-1.9]| = |-1.9 - (-2)| = 0.1$

$|-1.1 - [-1.1]| = |-1.1 - (-2)| = 0.9$

When $[x]$ is odd, e.g., $[1.5] = 1$, $[1.5 + 1] = 2$

$\therefore x - [x+1] = 1.5 - 2 = -0.5$

$\therefore |x - [x+1]| = [x+1] - x$ for $x \ge 0$

$\therefore$ acts like $y = -x$

For $x < 0$, e.g., -0.1, $[x] = -1$, $[x+1] = 0$

$\therefore |x - [x+1]| = |-0.1| = 0.1$

-0.9, $[x] = -1$, $[x+1] = 0$

$\therefore |x - [x+1]| = |-0.9| = 0.9$

$\therefore f(x):$