8.2 Finding Antiderivatives

Note Title 12/18/2014

11.

11. $f'(x) = \sqrt[3]{8x^2}$.

$f'(x) = 2x^{\frac{2}{3}} \quad \therefore \quad f(x) = 2\dfrac{x^{\frac{5}{3}}}{\frac{5}{3}}$

$f(x) = \frac{6}{5}x^{\frac{5}{3}} = \dfrac{6\sqrt[3]{x^5}}{5} + C$

22.

22. In each of the Exercises 17–21, what is the relation of the expression in the second parentheses to the expression inside the first parentheses? What formula for an antiderivative of $[f(x)]^k[f'(x)]$ do these exercises suggest? Verify your answer by differentiation.

$A_x^{-1}[f(x)]^k[f'(x)] = \dfrac{[f(x)]^{k+1}}{k+1}, \quad k \neq -1$

8.3 Generalized Power Formula for Antiderivatives

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9. **9.** $x\sqrt[7]{4x^2+15}$.

$$x(4x^2+15)^{\frac{1}{7}} = \frac{1}{8}(8x)(4x^2+15)^{\frac{1}{7}}$$

$$\therefore D_x^{-1} = \frac{\frac{1}{8}(4x^2+15)^{\frac{8}{7}}}{\frac{8}{7}} + C = \frac{7}{64}(4x^2+15)^{\frac{8}{7}} + C$$

14. **14.** $\dfrac{x-2}{x^3-6x^2+12x-8}$.

$$\begin{array}{r} x^2-4x+4 \\ x-2\overline{)\,x^3-6x^2+12x-8} \\ \underline{x^3-2x^2\quad\quad\quad} \\ -4x^2+12x-8 \\ \underline{-4x^2+8x\quad} \\ 4x-8 \\ \underline{4x-8} \\ 0 \end{array}$$

$$\therefore \frac{x-2}{x^3-6x^2+12x-8} = \frac{x-2}{x-2}\cdot\frac{1}{(x^2-4x+4)}$$

$$= \frac{1}{(x-2)^2} = (x-2)^{-2}$$

$$\therefore D_x^{-1} = \frac{(x-2)^{-1}}{-1} + C = -\frac{1}{x-2} + C, \quad x \neq 2$$

20. **20.** $(x^3 - 6)^3(2x^5 - 12x^2)$.

$$(x^3-6)^3(2x^5-12x^2) = (x^3-6)^3(2x^2)(x^3-6)$$

$$= (x^3-6)^4(2x^2) = \frac{2}{3}(x^3-6)^4(3x^2)$$

$$\therefore \Delta_x^{-1} = \frac{2}{3}\frac{(x^3-6)^5}{5} + C = \frac{2}{15}(x^3-6)^5 + C$$

8.4 Some Applications of Antiderivatives

Note Title 12/19/2014

1\. **1.** Find the equation of the curve whose slope at any point on it is equal to the square of the abscissa of that point, and which goes through the point (2, 1). Sketch the curve.

$$D_x y = x^2, \quad \therefore y = \frac{x^3}{3} + C$$

$$f(2) = 1 = \frac{8}{3} + C, \quad C = -\frac{5}{3}$$

$$\therefore y = \frac{x^3}{3} - \frac{5}{3}, \text{ or } 3y - x^3 + 5 = 0$$

8\. **8.** $a = 15\sqrt{t} + 8, \quad v_0 = -6, \quad x_0 = -44, \quad t = 4.$

$$D_t v = a = 15\sqrt{t} + 8 = 15t^{\frac{1}{2}} + 8$$

$$\therefore v(t) = \frac{15t^{\frac{3}{2}}}{\frac{3}{2}} + 8t + C$$

$$= 10t^{3/2} + 8t + C$$

$$\therefore v_0 = v(0) = C$$

$$\therefore v(t) = 10t^{3/2} + 8t + v_0$$

$$D_t x(t) = v(t) = 10t^{3/2} + 8t + v_0$$

$$\therefore x(t) = \frac{10t^{5/2}}{5/2} + \frac{8t^2}{2} + v_0 t + C$$

$$= 4t^{5/2} + 4t^2 + V_0 t + C$$

$$x(0) = x_0) = C$$

$$\therefore x(t) = 4t^{5/2} + 4t^2 + V_0 t + x_0$$

$$V_0 = -6, \therefore v(t) = 10t^{3/2} + 8t - 6$$

$$x_0 = -44, \therefore x(t) = 4t^{5/2} + 4t^2 - 6t - 44$$

At $t = 4$, $V(4) = 80 + 32 - 6 = 106$

$$x(4) = 4(32) + 4(16) - 24 - 44$$
$$= 128 + 64 - 68$$
$$= 124$$

15 **15.** On the moon, the acceleration due to gravity is approximately $\frac{1}{6}$ of what is on the earth. If a champion high jumper can clear 7 feet on earth, how high should he be able to jump on the moon?

Assume v_0 is the same on earth & moon.

$$v(t) = V_0 + at \quad (a = -32\,ft/sec^2)$$

$$\therefore d(t) = V_0 t + \frac{1}{2}at^2 + C, \; d(0) = 0, \text{ so } C = 0$$

$$\therefore V_f = V_0 + at, \; \frac{V_f - V_0}{a} = t$$

$$d = V_0\left(\frac{V_f - V_0}{a}\right) + \frac{1}{2}a\left(\frac{V_f - V_0}{a}\right)^2$$

$$= \frac{V_0 V_f - V_0^2}{a} + \frac{V_f^2 - 2V_0 V_f + V_0^2}{2a}$$

$$\therefore 2ad = 2V_0 V_f - 2V_0^2 + V_f^2 - 2V_0 V_f + V_0^2$$

$$= V_f^2 - V_0^2$$

At height of jump, $V_f = 0$

$$\therefore \frac{2a_e d_e}{2a_m d_m} = \frac{-V_0^2}{-V_0^2} = 1$$

$$\therefore a_e d_e = a_m d_m$$

$$a_e(7) = \frac{a_e}{6} d_m$$

$$42 = d_m$$

16. **16.** Compare the velocity at impact of an object dropped from a height of 200 feet above the moon's surface with the velocity at impact of an object dropped from the same height above the earth's surface.

From #15 above, $V_f^2 - V_i^2 = 2ad$

$$V_i = 0, \quad \therefore \frac{V_{f\,moon}^2}{V_{f\,earth}^2} = \frac{a_m}{a_e} = \frac{1}{6}$$

$$V_{f\,moon}^2 = \frac{V_{f\,earth}^2}{6}, \quad V_{f\,moon} = \frac{\sqrt{6}}{6} V_{f\,earth}$$