Chapter 2 - Motion in One Dimension

Note Title 10/11/2004

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17. $x(t) = 2.00 + 3.00t - t^2$

(a) Position at $t = 3.00$ sec.

$$x(3) = 2.00 + (3.00)(3.00) - (3.00)^2 = 2.00\,m$$

(b) $\text{Velocity} = \dfrac{dx(t)}{dt} = 3.00 - 2t$

$$= 3.00 - 2(3.00) = -3.00\,m/sec.$$

(c) $\text{Acceleration} = \dfrac{dv(t)}{dt} = -2$, or $-2.00\,m/sec^2$

21. $V_f^2 - V_i^2 = 2a(x_f - x_i)$

$$(10.97\,Km/sec)^2 - 0 = 2a(0.220 - 0)$$

$$a = \frac{(10.97\,Km/sec)^2}{2(0.220\,Km/sec)} = 273\,Km/sec^2$$

$$= 273 \times 10^3\,m/sec^2$$

30. (a) $x(t) = V_i t + \frac{1}{2}at^2 = 30t - \frac{1}{2}(2.00)t^2$

$$V(t) = 30 - 2.00t$$

(b) Max dist is when $V_f = 0$

$0 = 30 - 2.00t \;,\; t = 15 \text{ sec.}$

$\therefore x(15) = 30(15) - \frac{1}{2}(2.00)(15)^2$

$= 450 - 225 = 225 \text{ m}$

32. $\Delta x = V_i t + \frac{1}{2} a t^2 \;,\; V_f = V_i + at$

$62.4 = V_i(4.20) + \frac{1}{2}(-5.60)(4.20)^2$

$V_i = \dfrac{2(62.4) + (5.60)(4.20)^2}{2(4.20)} = 26.6 \text{ m/sec.}$

$V_f = 26.6 + (-5.60)(4.20) = 3.10 \text{ m/sec}$

33. $x_f - x_i = (V_f - at)t + \frac{1}{2} a t^2$

$= V_f t - a t^2 + \frac{1}{2} a t^2$

$= V_f t - \frac{1}{2} a t^2$

40. $D_{sue} = 30.0t - \frac{1}{2}(2.00)t^2$

$D_{van} = 155 + 5.00t$

Does t exist s.t. $D_{sue} = D_{van}$?

$30.0t - 1.00t^2 = 155 + 5.00t$

$0 = 155 - 25.0t + 1.00t^2$

$t = 11.4, 13.6$. So, will meet at 11.4 sec, and if not on same path, van would pass Sue at 13.6.

$\therefore t = 11.4$ will collide

$\therefore D_{van} = 155 + 5.00(11.4) = 212\ m$

If didn't collide (i.e., no real solution), would have to determine when Sue's speed reached 5 m/sec. That would be the closest approach.

42 (a) $4.00 = v_i(1.50) - \frac{1}{2}(9.8)(1.50)^2$, $V_i = 10.0\ m/sec$

(b) $V_f = v_i + at = 10.0 + (-9.8)(1.5)$

$= -4.68\ m/sec$

45. Dollar bill measures 15.5 cm long
$= 0.155\,m$
Determine where top of bill will be after 0.2 sec. If it is at or above midpoint (where fingers are), then will catch.

$$d = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(9.8)(0.2\ \text{sec})^2$$
$$= 0.196\,m = 19.6\ \text{cm}$$

$\therefore$ Top of bill will clearly be beyond midpoint and so will not catch bill.

46. Height of 1st ball : $h - \frac{1}{2}(9.8)t^2$
Height of 2nd ball : $v_i t - \frac{1}{2}(9.8)t^2$
At $h/2$, $h - \frac{1}{2}(9.8)t^2 = \frac{h}{2}$, so $\frac{h}{2} = \frac{1}{2}(9.8)t^2$,
or $t = \sqrt{\frac{h}{9.8}}$.

For second ball, $v_i t - \frac{1}{2}(9.8)t^2 = v_i t - \frac{h}{2} = \frac{h}{2}$,
so $v_i t = h$, $v_i = \frac{h}{t} = \frac{h}{\sqrt{\frac{h}{9.8}}} = \sqrt{9.8\,h}$ m/sec.

52. Velocity is $6.00t^2$ m/sec. Height $= 3.00(2)^3 = 24$ m

At 2.00 sec, $V = 24.0$ m/sec (upward)

$\therefore x_2 = x_1 + V_i t - \frac{1}{2} g t^2$

$0 = 24 + 24t - 4.9t^2$, $t = 5.75$ sec

53. (a)

$$a_x = \int_{t_i}^{t} J\,dt = a_{x_i} + Jt$$

$$V_x = \int_{t_i}^{t} a_x\,dt = \int_{t_i}^{t} (a_{x_i} + Jt)\,dt = V_{x_i} + a_{x_i} t + \frac{1}{2} J t^2$$

$$x = \int_{t_i}^{t} V_x\,dt = \int_{t_i}^{t} \left(V_{x_i} + a_{x_i} t + \frac{1}{2} J t^2\right) dt$$

$$= x_i + V_{x_i} t + \frac{1}{2} a_{x_i} t^2 + \frac{1}{6} J t^3$$

(b) $a_x^2 = (a_{x_i} + Jt)^2 = a_{x_i}^2 + 2a_{x_i} J t + J^2 t^2$

$V_x - V_{x_i} = a_{x_i} t + \frac{1}{2} J t^2$

$\therefore 2J(V_x - V_{x_i}) = 2J a_{x_i} t + J^2 t^2$

$\therefore a_x^2 = a_{xi}^2 + 2J(v_x - v_{xi})$

55. $a(t) = \frac{dv(t)}{dt} = -3.00\, v(t)^2$

$\therefore \frac{1}{V^2(t)} \frac{dv}{dt} = -3.00$

$\therefore \int_{t_1}^{t_2} \frac{1}{V(t)^2} \frac{dv(t)}{dt}\, dt = -\int_{t_1}^{t_2} 3.00\, dt$

Since $D_t\left(\frac{1}{V(t)}\right) = -\frac{1}{V(t)^2} \cdot \frac{dv(t)}{dt}$ (chain rule)

$-\frac{1}{V(t)} \Big|_{t_1}^{t_2} = -3(t_2 - t_1)$

$t_1 = 0,\ V(t_1) = 1.50\ m/sec,\ V(t_2) = \frac{1}{2} V(t_1)$

So $-\frac{1}{V(t_2)} + \frac{1}{V(t_1)} = -\frac{1}{\frac{1}{2}V(t_1)} + \frac{1}{V(t_1)} = -\frac{1}{V(t_1)}$

$= -\frac{1}{1.50} = -3t_2.\quad t_2 = \frac{1}{4.50}$

$= 0.222$ sec

56(a) $38.0\,m - (18.0\,m/sec)\Delta t = \frac{V_f^2 - V_i^2}{2a} = \frac{0-(18.0)^2}{(-4.50)2}$

$\therefore 38.0 - (18.0)\Delta t = 36.0$, $\Delta t = 0.111$ sec

$\therefore$ Max reaction time $= 0.111$ sec

b. $38.0 - 18.0(0.300) = \frac{V_f^2 - (18.0)^2}{2(-4.50)}$

$V_f = 5.53$ m/sec when hits deer.

57. From $V_f^2 - V_i^2 = 2a\Delta x$, $V_i = 0$ here, so

$V_f = \sqrt{2g\Delta x}$; $\Delta x = 15\,m$, so $V_f = \sqrt{30g}$

From $d = V_i\Delta t + \frac{1}{2}g(\Delta t)^2$, $d = 10\,m$ more,

$10 = \sqrt{30g}\,\Delta t + (4.9)(\Delta t)^2$, or

$4.9(\Delta t)^2 + 17.1(\Delta t) - 10 = 0$

$\Delta t = 0.510$ sec.

58. Dog hair acceleration = 7(0.132 mm)/week^2

59. $V_f^2 - V_i^2 = 2a\Delta x$, $V_f^2 = (80.0)^2 + 8(1000)$

V_f at rocket fuel burn out $= \sqrt{80^2 + 8000} = 120$ m/sec

$V_f = V_i + a\Delta t$, so $\Delta t = \dfrac{120 - 80}{4} = 10.0$ sec

∴ in air 5.50 sec during rocket burn

(a) At free fall, $x_f = x_i + v_i t - \frac{1}{2}gt^2$

$0 = 1000 + 120t - \frac{1}{2}(9.8)t^2$, $t = 31.1$ sec.

∴ $10.0 + 31.1 = 41.1$ sec in air above ground.

(b) $\dfrac{V_f^2 - V_i^2}{2g} = x_f - x_i$, $V_f = 0$, $V_i = 120$ m/sec, $x_i = 1000$

$x_f(\text{top}) = 1000 + \dfrac{0 - (120)^2}{2(-9.8)} = 1.73 \times 10^2$ m

(c). $V_f = V_i + a\Delta t = 120 + (-9.8)(31.1) = -185$ m/sec

62. $\Delta x_1 = \frac{1}{2}(v_i + v_m)t_1$, $\Delta x_2 = \frac{1}{2}(v_m + v_f)\,t_2$

where v_m = midpoint velocity
v_i = initial velocity $= 0 = v_f$ = final velocity
and $\Delta x_1 + \Delta x_2 = 1.00$ Km

$\therefore 1.00 \times 10^3 = \frac{1}{2} v_m t_1 + \frac{1}{2} v_m t_2$,

$\therefore \frac{2000}{v_m} = t_1 + t_2$, or $\frac{1}{v_m} = \frac{\Delta t}{2000}$

$v_m^2 - v_i^2 = 2a_1 \Delta x_1$, $v_f^2 - v_m^2 = 2a_2 \Delta x_2$, so

$\frac{v_m^2}{2(0.10)} + \frac{v_m^2}{2(0.50)} = 1000$, $\frac{1}{1000}\left[\frac{1}{0.20} + \frac{1}{1.0}\right] = \frac{1}{v_m^2}$

so $6.00 \times 10^{-3} = \frac{1}{v_m^2} = \left(\frac{\Delta t}{2 \times 10^3}\right)^2 = \frac{\Delta t^2}{4 \times 10^6}$

$\Delta t^2 = 24.0 \times 10^3$, $\Delta t = 1.55 \times 10^2 =$ 155 sec.

$v_m = \frac{2000}{\Delta t} = \frac{2000}{155} = 12.9$ m/sec

$v_m = v_i + a t_1$, $12.9 = (0.1)(t_1)$, $t_1 =$ 129 sec.

63. (a) Distance during acceleration:

$$\Delta_m = \tfrac{1}{2}(a_m)(2.00)^2 = 2a_m$$
$$\Delta_j = \tfrac{1}{2}(a_j)(3.00)^2 = 4.5a_j$$

Final velocities: $V_m = 2a_m$

$V_j = 3a_j$

$\therefore 100 = 2a_m + 2a_m t_1 \quad (\Delta_{acc} + \Delta_{const.\ vel.})$

$100 = 4.5a_j + 3a_j t_2 \quad (\Delta_{acc} + \Delta_{constant\ vel})$

$2\,sec + t_1 = 10.2, \quad t_1 = 8.2\,sec$
$3\,sec + t_2 = 10.2, \quad t_2 = 7.2\,sec$

$$\therefore a_m = \frac{100}{2+2t_1} = \frac{100}{2+2(8.2)} = 5.43\ m/sec^2$$

$$a_j = \frac{100}{4.5+3t_2} = \frac{100}{4.5+3(7.2)} = 3.83\ m/sec^2$$

(b) $V_m = 2a_m = 2(5.43) = 10.9\ m/sec$

$V_j = 3a_j = 3(3.83) = 11.5\ m/sec.$

(c) At 6 sec,

$M: 2a_m + 4V_m = 2(5.43) + 4(10.9) = 54.5m$

$J: 4.5a_j + 3V_m = 4.5(3.83) + 3(11.5) = 51.7m$

$\therefore$ at 6 sec, M ahead by 2.8m

64. Chest height $\approx 1.5m$, $g \approx 10\, m/sec^2$,

$V_f^2 = V_i^2 + 2g\Delta x = 2(10)(1.5) = 30$,

When ball hits pavement & slows to a stop, velocity $= 0$.

$\therefore 0 = 30 + 2a(0.01m)$,

$\frac{15}{0.01} = 150\, m/sec^2$

65. (a) $V_f = V_i + at$.

$\therefore 12m/sec = 0 + (3)\Delta t$, $\Delta t = 4sec$.

$\therefore 4sec + 5sec = 9sec$

Then stops: $0 = 12 + (-4.5)\Delta t$, $\Delta t = 2.67$

$\therefore 9 + 2.67 = 11.7$ secs.

Then accelerates to 18 m/sec: $18 = 0 + 3(\Delta t)$,
$\Delta t = 6$ sec. Cruises for 20 sec at 18 m/sec.
$\therefore$ 26 sec.
Slows down for 2.67 sec. $\therefore V_f = 18 + (-4.5)(2.67)$
$= 6.00$ m/sec. $26 + 2.67 = 28.7$ secs.
Cruises for 4 sec, $\therefore 28.7 + 4.0 = 32.7$ secs.
Stops: $0 = 6.00 + (-4.5)\Delta t$, $\Delta t = 1.33$ sec.
$\therefore 32.7 + 1.33 = 34$ sec.

$\therefore 11.7 + 34 =$ 45.7 sec.

6. 1st leg: $V_{ave} = \frac{12}{2} = 6$, for 4 sec. $\therefore$ 24 m
Cruise: $12(5) = 60$ m
Stop: $V_{ave} = 6$ m/sec, for 2.67 sec. $\therefore 6(2.67) = 16$ m
$\therefore 24 + 60 + 16 = 100$ m

2nd leg: $V_{ave} = 9.00$, at 6 sec, $\therefore 9(6) = 54$ m
Cruise: $20(18) = 360$ m
slows: $V_{ave} = \frac{18+6}{2} = 12$. $\therefore 12(2.67\text{ sec}) = 32$ m

Cruise: $4(6) = 24$ m
stops: $V_{ave} = 3.0$, for 1.33 sec, $\therefore 3(1.33) = 4$ m
$\therefore 54 + 360 + 32 + 24 + 4 = 474$

$\therefore 100 + 474 =$ 574 m

(c) $V_{ave} = 574 / 45.7\,sec = 12.6\ m/sec.$

(d) $2(574)/1.50 = 765\,sec.$

66. $d = \frac{1}{2} g t_1^2$; $t_1 = \sqrt{\frac{2d}{g}}$.

From sound bouncing up: $\frac{d}{336\ m/sec} = t_2$

$t_1 + t_2 = 2.40 = \sqrt{\frac{2d}{g}} + \frac{d}{336}$

$\therefore d + 336\sqrt{\frac{2}{g}}\sqrt{d} - 2.4(336) = 0$

Let $x^2 = d$ $\therefore x^2 + 152x - 806 = 0$, $x = 5.13$,

$\therefore x^2 = d = 26.3\ m$

(b) Neglecting sound, $d = \frac{1}{2} g t^2 = 4.9(2.4)^2$

$= 28.2\ m.$ $\therefore \frac{1.9}{26.3} = 7.2\%$

67. (a) $50 = (2\ m/sec)\Delta t + \frac{1}{2} g (\Delta t)^2$,

$4.9\Delta t^2 + 2\Delta t - 50 = 0$, $\Delta t = 3.0\ sec.$

(b) $50 = V_i(2.0\,sec) + \frac{1}{2} g (2.0)^2$, $V_i = 15.2\ m/sec.$

C. $V_f = v_i + g\cancel{t}$

$\therefore V_f = 2 + 9.8(3) = 31.4$ m/sec

$V_f = 15.2 + (9.8)(2) = 34.8$ m/sec

70. $h^2 + x^2 = l^2$

Let d = distance between height of food pack above level of hands.

$\therefore$ when d increases, l increases by the same amount. i.e., the hypotenuse lengthens by same amount that rope over limb shortens.

$$\therefore -\frac{dd}{dt} = \frac{dl}{dt}$$

$$\therefore 2h\frac{dh}{dt} + 2x\frac{dx}{dt} = 2l\frac{dl}{dt}$$

but h a constants, $\therefore x\, V_{boy} = l\frac{dd}{dt}$,

$$\frac{x}{l} V_{boy} = -\frac{dd}{dt} = \frac{x}{\sqrt{h^2+x^2}} V_{boy}$$

(b) Acceleration $= \dfrac{d}{dt}\left(x\,(h^2+x^2)^{-\frac{1}{2}}\,V_{boy}\right)$ Note: V_{boy} is a constant

$$= \left[V_{boy}\cdot\frac{dx}{dt}(h^2+x^2)^{-\frac{1}{2}} - \frac{1}{2}\,x\,V_{boy}\,(h^2+x^2)^{-\frac{3}{2}}\cdot(2x)\frac{dx}{dt}\right]$$

$$= \left[V_{boy}^2\cdot(h^2+x^2)^{-\frac{1}{2}} - x^2V_{boy}^2\,(h^2+x^2)^{-\frac{3}{2}}\right]$$

$$= V_{boy}^2\left[\frac{1}{\sqrt{h^2+x^2}} - x^2\frac{1}{\sqrt{(h^2+x^2)^3}}\right]$$

$$= V_{boy}^2\left[\frac{h^2+x^2}{\sqrt{(h^2+x^2)^3}} - \frac{x^2}{\sqrt{(h^2+x^2)^3}}\right]$$

$$= V_{boy}^2\,h^2\,(h^2+x^2)^{-\frac{3}{2}}$$

(c) $V_{pack} = 0$, $acc_{pack} = \dfrac{V_{boy}^2}{h}$

(d) As $x \to \infty$, $V_{pack} \to V_{boy}$, $acc_{pack} \to 0$

73. $x^2+y^2=L^2$, $y^2=L^2-x^2$, $y=\sqrt{L^2-x^2}$

When $\alpha=60^\circ$, $x=\frac{1}{2}L$

$$\frac{dy}{dt}=\frac{d}{dt}\left(L^2-x^2\right)^{\frac{1}{2}}=\frac{1}{2}\left(L^2-x^2\right)^{-\frac{1}{2}}\cdot\left(-2x\frac{dx}{dt}\right)$$

$$=-\frac{x}{\sqrt{L^2-x^2}}\cdot\frac{dx}{dt}$$

At $\alpha=60^\circ$, $\frac{\frac{L}{2}}{\sqrt{L^2-\frac{L^2}{4}}}=\frac{\frac{L}{2}}{\sqrt{\frac{3}{4}L^2}}$ $\frac{\sqrt{3}}{3}$

$$\therefore\frac{dy}{dt}=\frac{\sqrt{3}}{3}\frac{dx}{dt}=0.577v$$