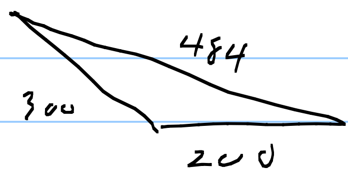
Chapter 3 - Vectors

Note Title 10/25/2004

7. (a). $c^2 = 200^2 + 300^2 - 2(200)(300)\cos 150°$
$= (483.65)^2$, $\therefore$ 484 Km

(b) $\dfrac{\sin 150°}{484} = \dfrac{\sin(x)}{300}$, $\sin x = 0.31$, $x = 18.1°$



$\therefore$ 18.1° north of west.

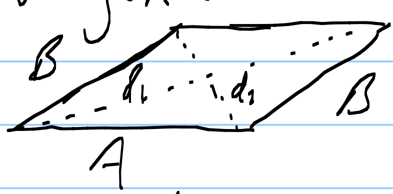
23. (a) Think in terms of unit vectors
1st displacement: $4.80\,\hat{i} + 0\,\hat{j} + 4.80\,\hat{k}$
2nd displacement: $0\,\hat{i} + 3.7\,\hat{j} + 3.7\,\hat{k}$

Resultant displacement: $4.80\,\hat{i} + 3.70\,\hat{j} + 8.50\,\hat{k}$

$\therefore \sqrt{4.8^2 + 3.7^2 + 8.5^2} = \sqrt{108.98} = 10.4$ cm

(b) $8.5 = 10.4\cos\theta$, $\theta = 35.5°$

53. Diagonals of a rhombus are perpendicular.



$2d_1 = 100(2d_2)$, so
$d_1 = 100\,d_2$.

Note: A = B

$\therefore \tan\theta = \dfrac{d_2}{d_1} = \dfrac{1}{100}$, $\theta = 0.573°$, so

$2\theta = 1.15°$

54. From #53, $\tan\frac{\theta}{2} = \frac{1}{n}$, $\frac{\theta}{2} = \arctan\frac{1}{n}$,

so $\theta = 2\arctan\left(\frac{1}{n}\right)$

60. (a) 1st step: $\frac{1}{2}(\vec{B}-\vec{A})$

2nd step: $\frac{1}{3}\left(\vec{C} - \frac{1}{2}(\vec{B}-\vec{A})\right)$

3rd step: $\frac{1}{4}\left[\vec{D} - \frac{1}{3}\left(\vec{C} - \frac{1}{2}(\vec{B}-\vec{A})\right)\right]$

4th step: $\frac{1}{5}\left[\vec{E} - \frac{1}{4}\left[\vec{D} - \frac{1}{3}\left(\vec{C} - \frac{1}{2}(\vec{B}-\vec{A})\right)\right]\right]$

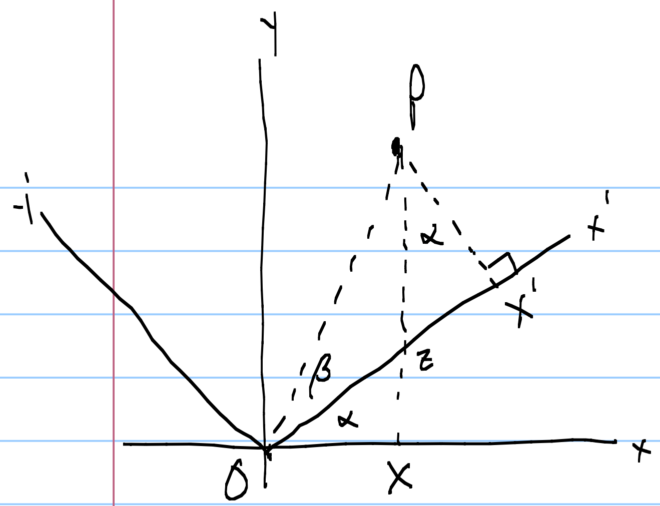
$= \frac{1}{5}\vec{E} - \frac{1}{20}\vec{D} + \frac{1}{60}\vec{C} - \frac{1}{120}\vec{B} + \frac{1}{120}\vec{A}$

$= (-14, 12) - (2, -1.5) + \left(-\frac{1}{6}, -\frac{1}{6}\right) - \left(0.5, \frac{2}{3}\right) + \left(\frac{1}{4}, -\frac{1}{6}\right)$

$= (-16.4, 12.5)$

63. Let P = length from origin to P

From similar Δ's, $m\angle x'Ox = m\angle xPx' = \alpha$



$$x' = OZ + ZX'$$

$$ZX' = PX'(\tan\alpha) = PX'\frac{\sin\alpha}{\cos\alpha}$$

But $PZ\cos\alpha = PX'$

$\therefore ZX' = PZ\sin\alpha$

$OZ\sin\alpha = ZX$, and $y = ZX + PZ$, so $PZ = y - ZX$

$$\therefore ZX' = PZ\sin\alpha = (y - ZX)\sin\alpha$$
$$= y\sin\alpha - ZX\sin\alpha$$
$$= y\sin\alpha - (X\tan\alpha)\cdot\sin\alpha$$

Also, $OZ\cos\alpha = X$, so $OZ = \frac{X}{\cos\alpha}$

$$\therefore X' = \frac{X}{\cos\alpha} + y\sin\alpha - X\frac{\sin\alpha}{\cos\alpha}.\sin\alpha$$

$$= X\left(\frac{1 - \sin\alpha\cdot\sin\alpha}{\cos\alpha}\right) + y\sin\alpha$$

$$= X\frac{(1 - \sin^2\alpha)}{\cos\alpha} + y\sin\alpha$$

$1 - \sin^2 = \cos^2$

$$\therefore x' = x\cos\alpha + y\sin\alpha$$

Now $y' = \rho x'$ and $y = \rho X = \rho z + zx$

$zx = x \tan\alpha$, $\rho z \cos\alpha = y'$

$$\therefore y = \frac{y'}{\cos\alpha} + x\tan\alpha$$

$$y\cos\alpha = y' + x\frac{\sin\alpha}{\cos\alpha}\cdot\cos\alpha$$

$$\therefore y' = -x\sin\alpha + y\cos\alpha$$