Chapter 4 - Motion in Two Dimensions

Note Title 11/15/2004

3. (a). $\vec{r} = (18.0t)\hat{i} + (4.00t - 4.90t^2)\hat{j}$

(b). $\frac{d\vec{r}}{dt} = (18.0)\hat{i} + (4.00 - 9.80t)\hat{j}$

(c) $\frac{d^2\vec{r}}{dt^2} = (-9.80)\hat{j}$

At $t = 3.00$ sec,

(d) $\vec{r} = (54.0)\hat{i} + (-32.1)\hat{j}$ meters

(e) $\vec{v} = (18.0)\hat{i} + (-25.4)\hat{j}$ m/sec

(f) $\vec{a} = (-9.80)\hat{j}$ m/sec^2

4. $\vec{r} = -5.00\sin(\omega t)\hat{i} + (4.00 - 5.00\cos(\omega t))\hat{j}$

(a) $\vec{v} = -5.00\,\omega\cos(\omega t)\hat{i} + 5.00\,\omega\sin(\omega t)\hat{j}$

$\vec{a} = 5.00\,\omega^2\sin(\omega t)\hat{i} + 5.00\,\omega^2\cos(\omega t)\hat{j}$

$\therefore$ at $t = 0$, $\vec{v} = -5.00\,\omega\hat{i}$

$\vec{a} = 5.00\,\omega^2\hat{j}$

(b) as above

(c) Circular motion, center at $(0, 4)$, radius $= 5.00$ m, starts at bottom of circle ($\vec{r} = -1.00\hat{j}$), moves clockwise

7. (a) $\vec{v_f} = \vec{v_i} + \vec{a}t$

$(20.0, -5.00) = (4.00, 1.00) + (a_x \cdot 20, a_y \cdot 20)$

$\therefore (16.0, -6.00) = (20a_x, 20a_y)$

$\vec{a} = (0.80, -0.30)\ m/sec^2$

(b) $\tan\theta = \frac{a_y}{a_x} = \frac{-0.30}{0.80}$, $\therefore \theta = -20.6°$
$= 360 - 20.6 = 339°$

(c) $\vec{V_f} = (4.00, 1.00) + (0.80 \times 25, -0.30 \times 25)$
$= (4.00 + 20, 1.00 - 7.5)$
$= 24.0\,\hat{i} - 6.50\,\hat{j}$ m/sec

$\tan\theta = \frac{-6.50}{24.0} = -0.271$, $\theta = -15.2°$
$= 360 - 15.2 = 345°$

$r_{fx} = 10.0 + 4.00(25) + \frac{1}{2}(0.80)(25)^2 = 360\ m$

$r_{fy} = -4.00 + 1.00(25) + \frac{1}{2}(-0.30)(25)^2 = -72.8\ m$

$\therefore \vec{r} = 360\,\hat{i} - 72.8\,\hat{j}$ meters

11. Range will be identical. Use $R = \frac{v_i^2 \sin 2\theta}{g}$, and v_i is the same

(a). $\sin 2(70) = \sin 140 = \sin(180-140) = \sin 40$

$\therefore \sin 40 = \sin 2\theta, \quad \theta = 20^\circ$

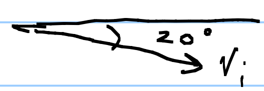
(b) $t = \dfrac{V_i \sin\theta}{g}$ = time to get to height

$\therefore$ For 70°: $\dfrac{2(25.0)\sin 70^\circ}{g} = 4.79$ secs

For 20°: $\dfrac{2(25.0)\sin 20^\circ}{g} = 1.74$ sec.

$\therefore$ Throw 2nd snowball 3.05 secs. later.

16. (a) $(8.00\,m/sec)(\cos 20^\circ)(3\,sec) = 22.5\,m$

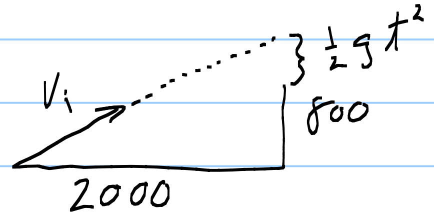


(b) $d = V_i t + \frac{1}{2}at^2 = (8.00)(\sin 20^\circ)(3\,sec) + (4.9)(9\,sec^2)$

$= 8.2 + 44.1 = 52.3\,m$

(c) $10 = (8)(\sin 20^\circ)t + 4.9t^2, \quad t = 1.2$ secs.

17.



$V_i = 1000\,m/sec.$

Time of flight $= \dfrac{x}{V_i \cos\theta}$

$$y = 800 = (V_i \sin\theta)t - \frac{1}{2}gt^2$$

$$= \frac{(V_i \sin\theta)(x)}{V_i \cos\theta} - \frac{1}{2}g\left(\frac{x}{V_i \cos\theta}\right)^2$$

$$\therefore 800 = 2000\tan\theta - \frac{(4.90)(2000)^2}{(1000)^2\cos^2\theta}$$

$$\therefore 800 = 2000\tan\theta - \frac{19.6}{\cos^2\theta}$$

$$= 2000\tan\theta - 19.6\sec^2\theta$$

$$= 2000\tan\theta - 19.6(1+\tan^2\theta)$$

$$= 2000\tan\theta - 19.6 - 19.6\tan^2\theta$$

$$\therefore 19.6\tan^2\theta - 2000\tan\theta + 819.6 = 0$$

$$\tan^2\theta - 102\tan\theta + 41.8 = 0$$

$$\tan\theta = 0.411, 102$$

$$\underline{\theta = 22.3^\circ}, \underline{89.4^\circ}$$

Note: $$y = x\tan\theta - \frac{gx^2}{2V_i^2}\cdot\frac{1}{\cos^2\theta}$$

$$= x\tan\theta - \frac{gx^2}{2V_i^2}(1+\tan^2\theta)$$

$$\therefore y = x\tan\theta - \frac{gx^2\tan^2\theta}{2V_i^2} - \frac{gx^2}{2V_i^2}$$

18. $$y = x\tan\theta - \frac{g x^2}{2v_i^2\cos^2\theta}$$

$$\frac{-b}{2a} = \frac{-\tan\theta}{\frac{-2g}{2v_i^2\cos^2\theta}} = \frac{v_i^2\cos^2\theta}{g}\cdot\tan\theta = \frac{v_i^2\sin\theta\cos\theta}{g}$$

$$\therefore x_{\text{max ht.}} = \frac{v_i^2\sin\theta\cos\theta}{g}$$

$$\text{Max range} = 2x_{\text{max ht.}} = \frac{v_i^2\, 2\sin\theta\cos\theta}{g} = \frac{v_i^2\sin 2\theta}{g}$$

19. (a) From $y = x\tan\theta - \frac{g x^2}{2v_i^2\cos^2\theta}$, $\theta = 53^\circ$, $v_i = 20.0\ \frac{m}{sec}$, $x = 36.0\ m$

$$\therefore y = (36.0)\tan 53^\circ - \frac{(9.8)(36.0)^2}{2(20.0)^2(\cos 53^\circ)^2}$$

$$= 3.93\ m.$$

$\therefore$ clears crossbar by $3.94 - 3.05 = .89\ m$

(b) $$x_{\text{max ht.}} = \frac{(20)^2\sin 53^\circ\cos 53^\circ}{g} = 19.6\ m$$

Since 36.0 m away, ball falling when approaching crossbar.

22. From speed of sound, $d = vt = (343\,m/sec)(3\,sec)$
$= 1029\ m$

At 40 m high, $40 = \frac{1}{2}gt^2$, $t = \sqrt{\frac{80}{9.8}} = 2.86\ sec$

$\therefore$ Traveled 1029 m in 2.86 sec = 360 m/sec

23. Assume "dunk" at peak.

Time flight going up: $1.85 - 1.02 = 0.83$
$d = \frac{1}{2}gt^2$, $t = 0.412$

Time flight going down: $1.85 - 0.90 = 0.95$
$d = \frac{1}{2}gt^2$, $t = 0.440$

(a) Total time flight = $0.412 + 0.440 = 0.852$ sec.

(b) Horiz. velocity = $\frac{2.80\ m}{\text{tot. time}} = \frac{2.8}{0.852} = 3.29\ m/sec.$

(c) Vertical velocity going up

$d = v_i t - \frac{1}{2}gt^2 = 0.83 = v_i(0.412) - (4.9)(0.412)^2$

$v_i = 4.03\ m/sec$

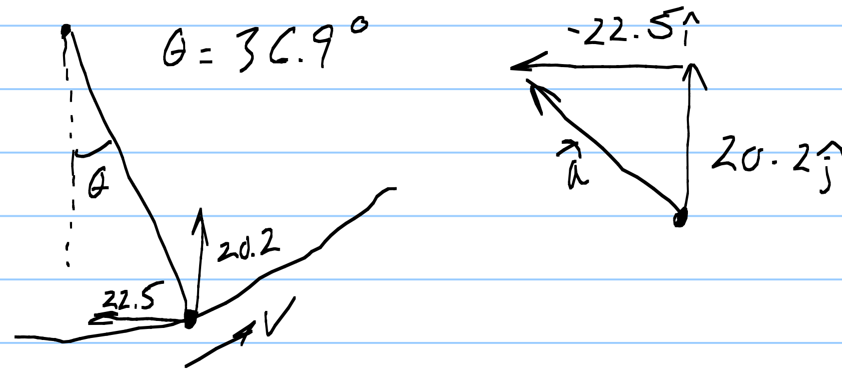
(d) $\tan\theta = \frac{V_v}{V_h} = \frac{4.03}{3.29} = 1.22$, $\theta = 50.8°$

(e) Up: $d = 2.50 - 1.20 = 1.3 = \frac{1}{2}gt^2$, $t = 0.515$

Down: $d = 2.50 - 0.70 = 1.8 = \frac{1}{2}gt^2$, $t = 0.606$

$\therefore t_{total} = 1.12$ sec.

35. (a)



(b) Radial acceleration = projection of $\hat{i}$ and $\hat{j}$ components along rope.

$\therefore 22.5 \cos(90 - 36.9) = 13.5$
$20.2 \cos(36.9) = 16.2$

$\therefore$ Total radial acc. $= 13.5 + 16.2 = 29.7$ m/sec²

(c) $a = \frac{v^2}{r}$, $v^2 = 29.7(1.5) = 6.67$ m/sec.,
36.9° above horizontal = same as θ

36. (a) $\vec{V}_f = \vec{V}_i + \vec{a}t$

Heather: $\vec{V}_f = \vec{0} + (3.00, -2.00)5$
$= (15.0, -10.0)$

Jill: $\vec{V}_f = (1.00, 3.00)5 = (5.00, 15.0)$

$\therefore \vec{V}_{Heather} - \vec{V}_{jill} = (10.0, -25.0)$

(b) $\vec{r}_f = r_i + \vec{V}_i t + \frac{1}{2}\vec{a}t^2$
$= \frac{1}{2}\vec{a}t^2$

$\therefore r_H = \frac{1}{2}(3.00, -2.00)25 = (37.5, -25.0)$

$r_j = \frac{1}{2}(1.00, 3.00)25 = (12.5, 37.5)$

$\therefore \sqrt{(37.5-12.5)^2 + (-25.0-37.5)^2} = 67.3 \text{ m}$

(c) $\vec{a}_H - \vec{a}_j = (15.0, -10.0) - (1.00, 3.00) = (14.0, -13.0)$

40. Very poorly worded.

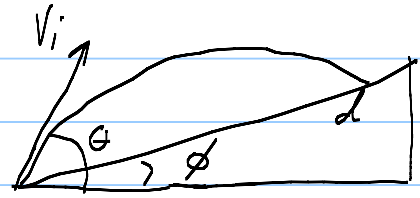
43. $V_{x\,train} = 10.0$ m/sec, since it can't have horizontal motion relative to earth.

$\therefore V_y = 10(\tan 60°) = 17.3$ m/sec

V_y same rel. to train & rel. to ground since rel. vel. of train = 0 in y direction.

$\therefore V_f^2 - V_i^2 = 2ad, \; V_i^2 = 2gd, \; d = \dfrac{v_i^2}{2g} = \dfrac{(17.3)^2}{2(9.8)} =$

15.3 m high.

47. (a) 

$d\cos\phi$ = horizontal distance of flight.

Time of flight $= \dfrac{d\cos\phi}{V_i\cos\theta}$

Vertical height at $d = d\sin\phi$, and also $(V_i\sin\theta)t - \frac{1}{2}gt^2$

$$\therefore d\sin\phi = (V_i\sin\theta)\frac{d\cos\phi}{V_i\cos\theta} - \frac{1}{2}g\left(\frac{d\cos\phi}{V_i\cos\theta}\right)^2$$

$$\therefore \sin\phi = \frac{\sin\theta\cos\phi}{\cos\theta} - \frac{1}{2}g\frac{d\cos^2\phi}{V_i^2\cos^2\theta}$$

$$\therefore -\sin\phi\cos\theta + \cos\phi\sin\theta = \frac{g}{2}\frac{d\cos^2\phi}{V_i^2\cos\theta}$$

$$\therefore \frac{2V_i^2\cos\theta\sin(\theta-\phi)}{g\cos^2\phi} = d$$

(b) Since ϕ is given, d is max when $f(\theta) = \cos\theta \sin(\theta - \phi)$ is a max.

$$f(\theta) = \sin(90-\theta)\sin(\theta-\phi)$$

$$f'(\theta) = \cos(90-\theta)\sin(\theta-\phi) + \sin(90-\theta)\cos(\theta-\phi)$$
$$= \sin[90-\theta-(\theta-\phi)]$$
$$= \sin(90-2\theta+\phi)$$

$f'(\theta) = 0$ when $90-2\theta+\phi = 0$, or $\theta = 45 + \frac{\phi}{2}$

$f''(\theta) = -2\cos(90-2\theta+\phi)$, so $f(\theta)$ is a max when $f'(\theta) = 0$.

$$\therefore \underline{\theta = 45^\circ + \phi/2}$$

use $\cos\theta = \sin(90-\theta)$

$$d_{max} = \frac{2v_i^2 \cos(45+\frac{\phi}{2})\sin(45-\frac{\phi}{2})}{g\cos^2\phi}$$

$$= \frac{2v_i^2 \sin(45-\frac{\phi}{2})\sin(45-\frac{\phi}{2})}{g\cos^2\phi} = \frac{2v_i^2\sin^2(\frac{90-\phi}{2})}{g\cos^2\phi}$$

$$= \frac{v_i^2[1-\cos(90-\phi)]}{g\cos^2\phi} = \underline{\frac{v_i^2(1-\sin\phi)}{g\cos^2\phi}}$$

60.(a) Equation of motion of ball is:

$$y = R + (\tan\theta)x - \left(\frac{g}{2v_i^2\cos^2\theta}\right)x^2$$

$$= R - \frac{gx^2}{2v_i^2}, \text{ since } \theta = 0°, x \geq 0$$

For the rock, $x^2 + y^2 = R^2$, or $y = \sqrt{R^2 - x^2}$, $0 \leq x \leq R$

As long as $y_{ball} > y_{rock}$ for all x, the ball will never hit the rock.

$$\therefore R - \frac{gx^2}{2v_i^2} > \sqrt{R^2 - x^2}, \quad R - \sqrt{R^2 - x^2} > \frac{gx^2}{2v_i^2},$$

$$v_i^2 > \frac{gx^2}{2(R - \sqrt{R^2 - x^2})}, \text{ for all } x \text{ s.t. } 0 \leq x \leq R$$

$$= \frac{gx^2}{2(R - \sqrt{R^2 - x^2})} \cdot \frac{R + \sqrt{R^2 - x^2}}{(R + \sqrt{R^2 - x^2})}$$

$$= \frac{gx^2(R + \sqrt{R^2 - x^2})}{2(R^2 - R^2 + x^2)} = \frac{g(R + \sqrt{R^2 - x^2})}{2}$$

$$\therefore v_i^2 > \frac{g(R + \sqrt{R^2 - x^2})}{2}, \quad 0 \leq x \leq R$$

$\frac{g\left(R+\sqrt{R^2-x^2}\right)}{2}$ is a max when $x=0$.

$\therefore\ V_i^2 > gR$, $V_i > \sqrt{gR}$

$\therefore$ if $V_i > \sqrt{gR}$, ball won't hit rock.

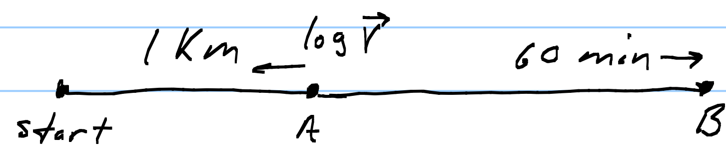
(b) if $V_i = \sqrt{gR}$, Then $y = R - \frac{g x^2}{2 V_i^2}$,

$$y = R - \frac{g x^2}{2 g R} = R - \frac{x^2}{2R}.$$

Ball hits ground at $y=0$. $\therefore R = \frac{x^2}{2R}$,

$x^2 = 2R^2$, $x = \sqrt{2}\, R$

64.



Let T = time to travel by boat from B to start, in minutes.

$\therefore$ Log travels 1 Km in $T+60$ minutes.

$\therefore V_{river} = \frac{1000}{T+60}$ m/min, or $VT + 60V = 1000$

Let R = speed of boat in still water.

$\therefore \quad 1000 + (R-V)60 = (R+V)T$

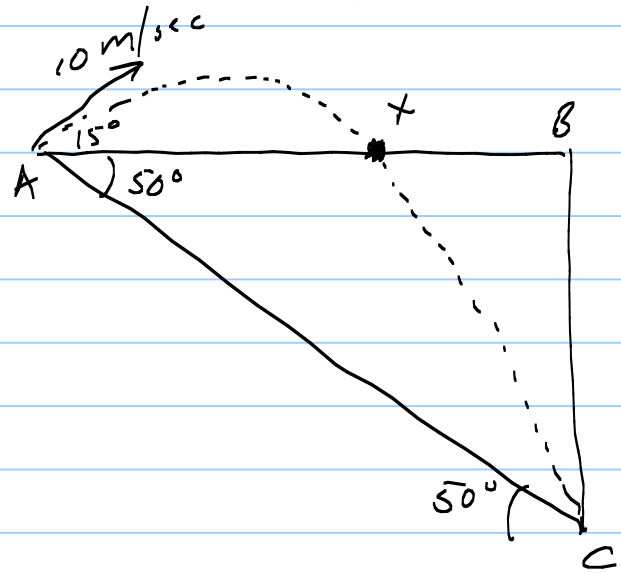
Substituting for 1000,

$$VT + 60V + 60R - 60V = RT + VT$$

$\therefore \quad 60R = RT, \quad T = 60\,\text{min}$

$\therefore \quad V_{river} = \dfrac{1000}{60+60} = 8.33\ \text{m/min}.$

Note that speed of boat $= R$ can be anything.

17. 

Let T = time of flight in secs.

$\therefore \quad AB = (10\cos 15°)\,T$

$BC = AB \tan 50°$
$= (10\cos 15°)(\tan 50°)\,T$

$\overline{BC}$ can also be computed from $vt + \frac{1}{2}at^2$

Vertical speed at takeoff = $10(\sin 15°)$
= vertical speed at point X.

Also, time from liftoff to height is from:
$V_f = V_i + at$, or $0 = 10(\sin 15°) - gt$, or

$$t = \frac{10 \sin 15°}{g} \therefore T_{AX} = \frac{20 \sin 15°}{g} = 0.528$$

$$\therefore BC = (10 \sin 15°)(T - T_{AX}) + \tfrac{1}{2} g (T - T_{AX})^2$$

$$\therefore 10(\cos 15°)(\tan 50°)T = 10(\sin 15°)(T - T_{AX}) + \tfrac{1}{2} g (T - T_{AX})^2$$

$$11.5T = 2.59T - 1.37 + 4.9(T - 0.528)^2$$

$$0 = -1.37 - 8.91T + 4.9(T - 0.528)^2$$

$$T = 2.87 \text{ sec}$$

$$\therefore AB = 10(\cos 15°)(2.87) = 27.7 \text{ m}$$

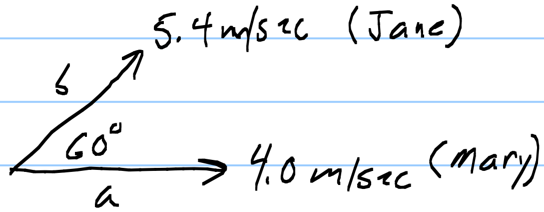
$$BC = AB \tan 50° = 33.0 \text{ m}$$

$$AC = AB / \cos 50° = 43.1 \text{ m}$$

(6) Horiz veloc $= 10 \cos 15° = 9.66$ m/sec
Vert. veloc. $= 10 \sin 15° - 9.8(2.87) = -25.5$ m/sec
$\therefore \vec{V} = 9.66\hat{i} - 25.5\hat{j}$ m/sec.

68. Use law of cosines : $c^2 = a^2 + b^2 - 2ab\cos\theta$

(a)



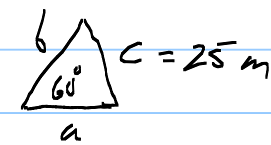
Let T = time (secs) of travel.

$\therefore (25.0)^2 = (4.0T)^2 + (5.4T)^2 - 2(4.0T)(5.4T)(\frac{1}{2})$

$625 = 16T^2 + 29.2T^2 - 21.6T^2$

$T = 5.15$ sec

(b) Velocity of Jane relative to Mary



Side b : $(5.40 \text{ m/sec})(5.15 \text{ sec}) = 27.8 \text{ m}$

side a : $(4.00 \text{ m/sec})(5.15 \text{ sec}) = 20.6 \text{ m}$

$\therefore b^2 = a^2 + c^2 - 2ac\cos\angle a\text{-}c$

$27.8^2 = 20.6^2 + 25^2 - 2(20.6)(25.0)\cos\theta, \theta = 74.4°$

$\therefore m\angle a\text{-}c = 180 - 74.4 = 105.6°$ N of East

Using similar triangles (distance & velocity), Velocity vector from Mary to Jane

$\frac{c}{a} = \frac{\text{rel. veloc.}}{V_{mary}}, \quad \frac{25}{20.6} = \frac{V}{4.00}, \quad V = 4.85$

$\therefore$ Jane rel. to Mary : 4.85 m/sec, 105.6° N of East.

70. Equation of motion is (see problem #17):

$$y = x\tan\theta - \frac{gx^2}{2V_i^2}(1+\tan^2\theta)$$

Given $x = 2500$, we want $y > 1800$, so what range of θ's satisfies these conditions, and with those θ's, what is associated ranges?

$$\therefore\ 2500\tan\theta - \frac{9.8(2500)^2}{2(250)^2}(1+\tan^2\theta)$$

$$= 2500\tan\theta - 490(1+\tan^2\theta)$$

Let $T = \tan\theta$

$$2500T - 490 - 490T^2 \geq 1800$$

$$490T^2 - 2500T + 2290 \leq 0$$

$$1.1967 \leq T \leq 3.9054$$

$$1.1967 \leq \tan\theta \leq 3.9054$$

$$50.1^\circ \leq \theta \leq 75.6^\circ$$

Range must be ≥ 2800

$$\therefore\ \frac{V_i^2 \sin 2\theta}{g} = \frac{(250)^2 \sin 2\theta}{g} = 6377.6\sin 2\theta$$

$\therefore$ $6377.6 \sin 2\theta \geq 2800$, $\sin 2\theta \geq 0.439$

$\therefore$ $26° \leq 2\theta \leq 180 - 26 = 154°$

$\therefore$ $13° \leq \theta \leq 77°$

$\therefore$ Intersection of θ ranges (clears hill top and clears eastern shore) is:

$$50.1° \leq \theta \leq 75.6°$$

As all these are $> 45°$, $\theta = 75.6°$ will give shortest range. That range will be

$$\frac{250^2 \sin(2 \cdot 75.6°)}{9.8} = 3072 \text{ meters}$$

$\therefore$ If ship is within $3072 - 2800 = 272$m of shoreline, it will not be hit.

This all assumes mountain outline is within triangle defined by peak and 300m eastern shoreline.