Chapter 5 -The Laws of Motion

Note Title 12/12/2004

5. $V_f^2 - V_i^2 = 2ad$, so $(320)^2 - 0^2 = 2a(0.820)$,

$$a = \frac{320^2}{2(0.82)} = 6.24 \times 10^4 \text{ m/sec}^2$$

$\therefore F = (0.005)(6.24 \times 10^4) = 312$ N

17. Assume mass of man $\simeq 100$ Kg, $g = 10$ m/sec^2

$\therefore F = 10^3$ N.

Mass of earth $= 6 \times 10^{24}$ Kg $\approx 10^{25}$ Kg.

$$\therefore a_e \simeq \frac{10^3 \text{ N}}{10^{25} \text{ Kg}} = 10^{-22} \text{ m/sec}^2$$

(b) $d = \frac{1}{2}at^2$. Assume $t \approx \frac{1}{2}$ sec (chair to floor flight)

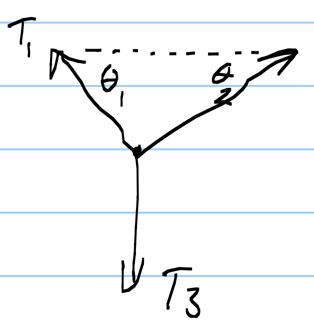
$$\therefore d \simeq \frac{1}{2}(10^{-22})\left(\frac{1}{2}\right)^2 \simeq 10^{-23} \text{ m}$$

22. $x''(t) = 10$, $y''(t) = 18t$. $\therefore$ at $t = 2$ sec,

$a_x = 10$, $a_y = 36$ $|a| = \sqrt{10^2 + 36^2} = 37.4$ m/sec^2

$\therefore F = (3.00 \text{ kg})(37.4 \text{ m/sec}^2) = 112$ N

24.



$$-T_1 \cos\theta_1 + T_2 \cos\theta_2 = 0$$

$$T_1 \sin\theta_1 + T_2 \sin\theta_2 - T_3 = 0, \quad T_3 = F_g$$

$$T_2 = \frac{T_1 \cos\theta_1}{\cos\theta_2} \quad \therefore \; T_1 \sin\theta_1 + T_1 \frac{\cos\theta_1}{\cos\theta_2} \cdot \sin\theta_2 = F_g$$

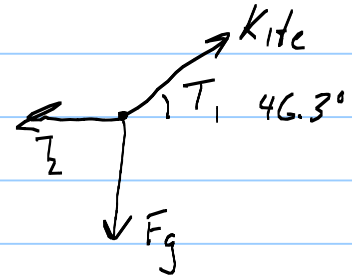
$$\therefore \; T_1 \sin\theta_1 \cos\theta_2 + T_1 \cos\theta_1 \sin\theta_2 = F_g \cos\theta_2$$

But $\sin(\theta_1 + \theta_2) = \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2$

$$\therefore \; T_1 \sin(\theta_1 + \theta_2) = F_g \cos\theta_2 ,$$

$$T_1 = \frac{F_g \cos\theta_2}{\sin(\theta_1 + \theta_2)}$$

26. (b).

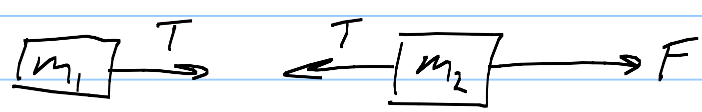


$$F_g = 0.132\,(9.8) = 1.29\ N$$

$$T_1 \sin 46.3^\circ = 1.29\ N$$

$$\therefore \; T_1 = \frac{1.29}{\sin 46.3^\circ} = 1.79\ N$$

35.



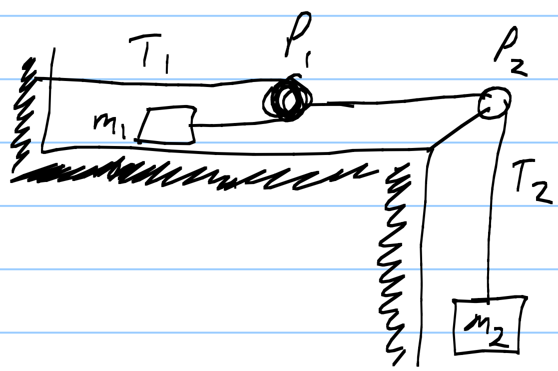
Both m_1 and m_2 accelerate the same.

$$\therefore \; a = F/(m_1 + m_2) , \quad \therefore \; T = m_1 a = \frac{m_1}{m_1 + m_2} F$$

Note That $F - T = m_2 a = \dfrac{m_2}{m_1 + m_2} F$

$\therefore \; T = F - \dfrac{m_2}{m_1 + m_2} F = \dfrac{m_1}{m_1 + m_2} F$, as above.

38. (a)

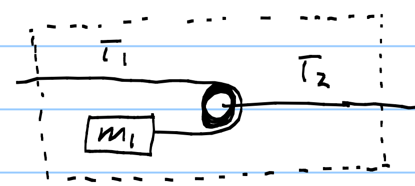


For each Δx That P_1 moves to The right, m_1 moves closer to P_1 by Δx.
$\therefore$ m_1 moves $2\Delta x$ to The right with respect to The table.

$\therefore \; a_1 = 2 a_2$

(b) (1) $T_1 = m_1 a_1$ (2) $m_2 g - T_2 = m_2 a_2$

(3) $T_2 - 2T_1 = m_1 a_2$ using:



i.e., The mass inside the dotted box is m_1, and The free body (The dotted box) accelerates at a_2.

$\therefore$ Adding (2) and (3): $m_2 g - 2T_1 = (m_1 + m_2) a_2$

Substituting for T_1 using (1), and $a_1 = 2a_2$,

$$m_2 g - 2(m_1 a_1) = (m_1 + m_2) a_2$$

$$m_2 g - 2(m_1)(2a_2) = (m_1 + m_2) a_2$$

$$m_2 g = 4 m_1 a_2 + (m_1 + m_2) a_2$$

$$m_2 g = (5 m_1 + m_2) a_2$$

$$\therefore \quad a_2 = \frac{m_2}{5m_1 + m_2} g$$

$$a_1 = \frac{2 m_2}{5 m_1 + m_2} g$$

$$T_1 = m_1 a_1 = \frac{2 m_1 m_2}{5 m_1 + m_2} g$$

$$T_2 = m_2 g - m_2 a_2$$

$$= m_2 g - \frac{m_2^2 g}{5m_1 + m_2} = g\left(\frac{5 m_1 m_2 + m_2^2 - m_2^2}{5 m_1 + m_2}\right)$$

$$= \frac{5 m_1 m_2}{5 m_1 + m_2} g$$

40. $F_s = -\mu_s N$. Force of earth on runner is also $\mu_s N$, where $N = mg$.

$\therefore$ For runner, $\mu_s mg = ma$, so $\mu_s g = a$,

so runner's mass is irrelevant.

41. $F = \mu_s N = 75.0 = \mu_s mg = \mu_s (25.0)(9.8)$

$$\therefore \mu_s = \frac{75.0}{(25.0)(9.8)} = 0.306$$

$60 - F_K = 0$, $F_K = \mu_K N = 60.0$

$$\mu_K = \frac{60.0}{(25.0)(9.8)} = 0.245$$

43. (a) $F_K = -\mu_K N = -\mu_K mg = ma$, so $a = -\mu_K g$

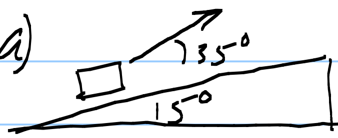
$v_f^2 - v_i^2 = 2ad$, $\therefore -v_i^2 = 2(-\mu_K g)d$

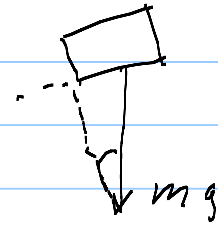
$$\therefore d = \frac{v_i^2}{2\mu_K g} = \frac{\left[(50\,\text{mi/hr})(1.61\times10^3\,\text{m/mi})\frac{1\,\text{hr}}{3600\,\text{sec}}\right]^2}{2(0.1)(9.8)}$$

$$= 255\,\text{m}$$

(b) $255\left(\frac{0.1}{0.6}\right) = 42.5\text{ m}$

47. (a)





Force from gravity $= mg\sin 15°$

Force from rope $= 25.0(\cos[35-15]) = 25.0\cos 20°$

$\therefore 25\cos(20°) - mg\sin(15°) - F_f = 0$ [1]

For the normal forces, gravity: $mg\cos(15°)$
rope: $25.0(\sin 20°)$

$\therefore N + 25.0(\sin 20°) - mg(\cos 15°) = 0$ [2]

Also, $F_f = \mu_k N$ [3]

From [2], $N = mg(\cos 15°) - 25.0(\sin 20°)$

From [1] and [3], $\mu_k N = 25.0(\cos 20°) - mg(\sin 15°)$

$$\therefore \mu_k = \frac{25.0(\cos 20°) - mg(\sin 15°)}{mg(\cos 15°) - 25.0(\sin 20°)}$$

$= 0.161$, using $mg = 60.0\text{ N}$

(b) $F_f = \mu_k N = (0.161)\,mg$

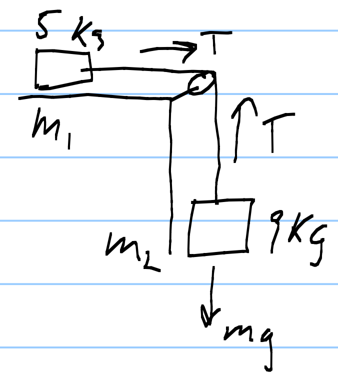
$F_g = mg(\sin 15^\circ)$

$ma = mg(\sin 15^\circ) - (0.161)\,mg$

$\therefore a = g(\sin 15^\circ) - (0.161)\,g$

$= 0.957 \ m/sec^2$

49.



$T - F_f = 5(a)$ [1]

$m_2 g - T = 9(a)$ [2]

$9g - F_f = 14(a)$, $F_f = \mu m_1 g$

$\therefore 9g - \mu 5g = 14a$, $a = \dfrac{9g - (0.200)(5)(g)}{14}$

From [2], $T = m_2 g - 9a$

$= 9g - 9\left[\dfrac{9(g) - (0.200)(5)(g)}{14}\right]$

$= 37.8 \ N$

50.

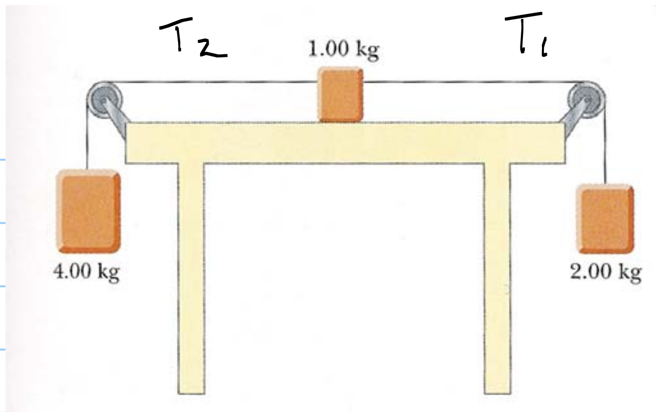


Figure P5.50

Assume acceleration is to left for m_1

$F_f = \mu_K N$, $N = m_1 g$

so $F_f = \mu_K m_1 g$

(a)

For m_4: $m_4 g - T_2 = m_4 a$ [1]

m_2: $T_1 - m_2 g = m_2 a$ [2]

m_1: $T_2 - T_1 - F_f = m_1 a$, $T_2 - T_1 - \mu_K m_1 g = m_1 a$ [3]

Adding [1], [2], and [3],

$$m_4 g - m_2 g - \mu_K m_1 g = (m_1 + m_2 + m_4) a$$

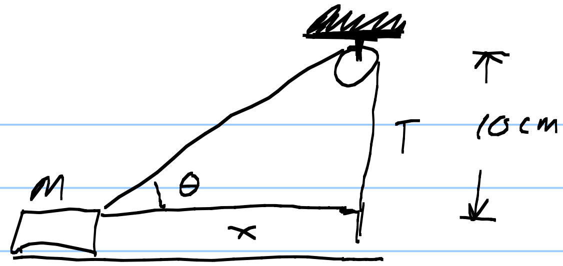
$$a = \frac{m_4 - m_2 - \mu_K m_1}{m_1 + m_2 + m_4} g = \frac{4 - 2 - 0.350(1)}{7} g$$

$$= 2.31 \text{ m/sec}^2$$

(b) From [1], $4g - T_2 = 4(2.31)$, $T_2 = 30.0$ N

From [2], $T_1 = 2(2.31 + 9.8) = 24.2$ N

52.



$M = 2.20$ Kg $T = 10.0$ N

$\mu_k = 0.400$

Forces on M (horiz): $T\cos\theta - F_k$

$$\cos\theta = \frac{x}{\sqrt{x^2+0.1^2}} = \frac{x}{\sqrt{x^2+0.01}} \qquad F_k = \mu_k(Mg - T\sin\theta)$$

$$\sin\theta = \frac{0.10}{\sqrt{x^2+0.01}}$$

$$\therefore F_x = \frac{Tx}{\sqrt{x^2+0.01}} - \mu_k Mg - \frac{\mu_k T(0.100)}{\sqrt{x^2+0.01}}$$

(a) When $x = 0.400$ m,

$$F_x = \frac{(10)(0.4)}{\sqrt{0.16+.01}} - 0.4(2.2)(9.8) - \frac{(0.4)(10)(0.1)}{\sqrt{0.16+0.01}}$$

$$= 9.70 - 8.62 - 0.97 = 0.107 \text{ N}$$

$$\therefore a = \frac{F_x}{m} = \frac{0.107}{2.2} = 0.236 \text{ m/sec}^2$$

(b) Accel. = 0 when $F_x = 0$

$$\therefore\ 0 = \frac{Tx}{\sqrt{x^2+0.01}} - \mu_k mg - \frac{\mu_k T(0.10)}{\sqrt{x^2+0.01}}$$

$$\mu_k mg = \frac{Tx - \mu_k T(0.10)}{\sqrt{x^2+0.01}}$$

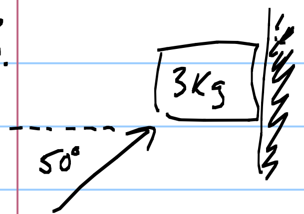
$$\frac{\mu_k mg}{T} = \frac{x - \mu_k(0.10)}{\sqrt{x^2+0.01}}$$

$$\left(\frac{\mu_k mg}{T}\right)^2 (x^2 + 0.01) = [x - \mu_k(0.1)]^2$$

$$\left[\frac{(0.4)(2.2)(9.8)}{10}\right]^2 (x^2+0.01) = [x - 0.04]^2$$

$$0.744x^2 = (x-0.04)^2,$$

$x = 0.29,\ 0.0215$. Reaches $x = 0.29\,m$ first.

53. 

$P(\sin 50°) - 3g - F_f$ are forces on m.

$\therefore$ When $P\sin 50° - 3g > F_f$, block will move up.

If block slides down, forces on m are $P\sin 50° - 3g + F_f$

$F_f = \mu_s N$ (a contact force)

$= \mu_s P\cos 50°$

$\therefore$ Prevent sliding up, want $F_y - F_g \le F_f$

$$P\sin 50° - 3g \le \mu_s P\cos 50°$$

$$P(\sin 50° - \mu_s \cos 50°) \le 3g$$

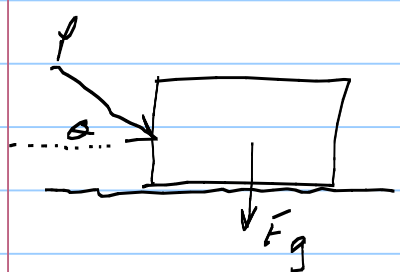
$$P \le \frac{3g}{\sin 50° - (.25\cos 50°)} = 48.6\ N$$

Prevent falling, want $F_y + F_f \ge F_g$

$\therefore P\sin 50° + \mu_s P\cos 50° \ge 3g$

$$P \geq \frac{3(9.8)}{\sin 50^\circ + (.25)(\cos 50^\circ)} = 31.7\,N$$

$\therefore 31.7\,N \leq P \leq 48.6\,N$

61. 

Assume P acts on center of mass, and so P exerts a force opposite the normal force.

normal force: $F_g + P\sin\theta$

frictional force: $= \mu_s N = \mu_s F_g + \mu_s P\sin\theta$

$P\cos\theta$ must match the static frictional force.

$$\therefore \quad P\cos\theta = \mu_s F_g + \mu_s P\sin\theta$$

$$P\cos\theta - \mu_s P\sin\theta = \mu_s F_g$$

$$P(\cos\theta - \mu_s \sin\theta) = \mu_s F_g$$

$$P = \mu_s F_g \cdot \frac{1}{\cos\theta - \mu_s \sin\theta} = \mu_s F_g \frac{\frac{1}{\cos\theta}}{1 - \mu_s \tan\theta}$$

$$= \mu_s F_g \sec\theta \,(1 - \mu_s \tan\theta)^{-1}$$

63. From #61, and using P directed above horizontal,

$$P\cos\theta = \mu_s F_g - \mu_s P\sin\theta,$$

$$P(\theta) = \mu_s F_g (\cos\theta + \mu_s \sin\theta)^{-1}$$

$P(\theta)$ is a minimum when $(\cos\theta + \mu_s \sin\theta)$ is a max.

Let $f(\theta) = \cos\theta + \mu_s \sin\theta$

$\therefore f'(\theta) = -\sin\theta + \mu_s \cos\theta$

$f''(\theta) = -\cos\theta - \mu_s \sin\theta$

$f'(\theta) = 0 = -\sin\theta + \mu_s \cos\theta$, $\tan\theta = \mu_s$

$\theta = \arctan(\mu_s) = \arctan(0.35) = 19.3°$

since $f''(19.3°) < 0$, $f'(19.3)$ is a max.

$\therefore$ Tension at a minimum when $\theta = 19.3°$

(b) $$\text{Tension} = \frac{\mu_s F_g}{\cos\theta + \mu_s \sin\theta} = \frac{(0.35)(1.3)(9.8)}{\cos(19.3) + 0.35(\sin 19.3)}$$

$$= 4.21\ N$$

65. (a) Use $d = \frac{1}{2}at^2$ $\quad a_2 = (F - F_f)/m_2$

$$F_f = \mu_k N = (0.3)(2.0)(9.8)$$

$$\therefore\ a_2 = \frac{10 - (0.3)(2.0)(9.8)}{2.0} = 2.06\ m/sec^2$$

This is acceleration relative to earth

For 8kg block, F_f is only force moving it.

$$\therefore\ a_8 = F_f / 8.0 = \frac{(0.3)(2.0)(9.8)}{8.0} = 0.735$$

$\therefore$ a_2 acceleration relative to 8kg block is

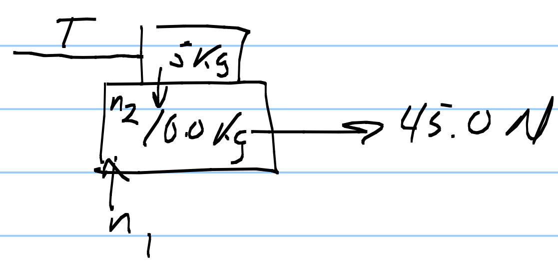
$$2.06 - 0.735 = 1.325\ m/sec^2$$

$$\therefore\ d = 3.0 = \tfrac{1}{2} a t^2 = \tfrac{1}{2}(1.325)t^2,$$

$$t = 2.13\ \text{secs.}$$

(b). $d = \frac{1}{2} a t^2 = \frac{1}{2}(0.735)(2.13)^2 = 1.67\ m$

68. (a)



$$45 - F_1 - F_2 = 10.0\,a$$

$$F_1 = \mu_K N_1 = (0.2)(15)(9.8)$$

$$F_2 = \mu_K N_2 = (0.2)(5)(9.8)$$

$$\therefore\ a = \frac{45 - (0.2)(15)(9.8) - (0.2)(5)(9.8)}{10.0} = 0.58\ m/sec^2$$

(b) $F_2 = T = \mu_k N = (0.2)(5.0)(9.8) = 9.8\,N$

77. (a) As in problem # 40, force of earth on car $= \mu N$.
$\therefore \mu_s mg = ma$, so a independent of m.

Using $d = \frac{1}{2}at^2$, $a = 2d/t^2$

$$\therefore a = \frac{2(0.25\,mi)(1.609\times10^3\,m/mi)}{(4.96)^2} = 32.7\,m/sec^2$$

$$\therefore \mu_s = \frac{a}{g} = \frac{32.7}{9.8} = 3.34$$

(b) Rotation of wheels is a torque via the axle. The axle is not completely frictionless, and so when the tires rotate, an equal but opposite rotation occurs on rest of car. $\therefore$ The front wheels lift.
In (a) front wheels barely lifted, which means you were at a max for rotational friction. More power means the wheels in front will lift more, or the tires will skid as force is greater than static friction.