Chapter 6 - Circular Motion

Note Title 1/10/2005

3. $T_{max} = mg = (25)(9.8)$

For circular motion $F = T = m\frac{v^2}{r}$

$\therefore m\frac{v^2}{r} \leq T_{max} = (25)(9.8)$

$v^2 \leq \frac{(25.0)(9.8)(0.8)}{3.0}$, $v \leq 8.08$ m/sec.

9. $F = m\frac{v^2}{r} = \frac{m(0.50)^2}{(0.30)}$, $N = mg$, $F = \mu mg$

$\therefore \mu mg = m\frac{(0.50)^2}{0.30}$, $\mu = \frac{(0.50)^2}{(0.30)g} = 0.0850$

15. $m\frac{v^2}{r} + mg = 85.0\left(\frac{8.0^2}{10.0} + 9.8\right) = 1377\ N > 1{,}000\ N.$

$\therefore$ The rope breaks.

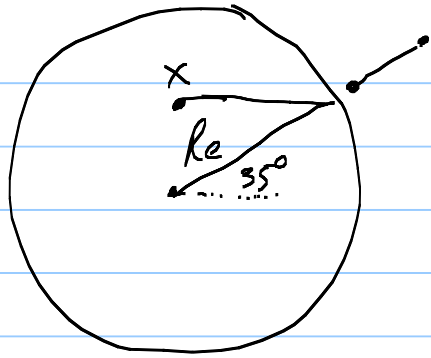
25. (a) $a = g\tan\theta$, $3.00 = 9.8(\tan\theta)$, $\theta = \arctan\left(\frac{3.0}{9.8}\right)$

$\theta = 17.0^\circ$

(b) 

$T\cos\theta = mg$, $T = \frac{(0.5)(9.8)}{\cos 17^\circ} = 5.12\ N$

29.



R_e = radius of earth

$R_e \cos 35°$ = radius of orbit of bob.

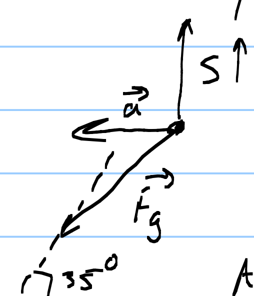
$V_{bob} = \dfrac{2\pi R_e \cos\theta}{T}$, $T = 86{,}400$ sec

$\therefore \dfrac{V_{bob}^2}{R_e \cos\theta}$ = acceleration of bob toward point X.

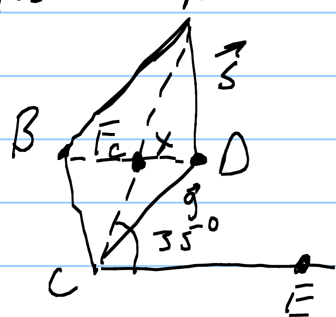
$= \dfrac{4\pi^2 R_e \cos\theta}{T^2}$

Note: R_e is a good approx. of bob's radius of orbit.

Acceleration from gravity is $\vec{g}$ and toward center of earth.
The only other force is from the rope ($\vec{s}$)



$\vec{a}$ is the centripetal acceleration.
The deflection angle (toward equator). $\vec{F_s} + \vec{F_g} = \vec{F_c}$, where $\vec{F_c}$ is centripetal force, $\vec{F_s}$ = rope.



Deflection angle is $\angle CAD$

A good approximation of $\vec{S}$ is $m\vec{g}$, since the deflection angle is bound to be very small. $\therefore$ acceleration vector of $\vec{S}$ is effectively of magnitude g.

Also, $\square ABCD$ is a rhombus.

$\therefore \overline{AC} \perp \overline{BD}$

$$\therefore m\angle CAD = \arcsin\left(\frac{XD}{AD}\right)$$

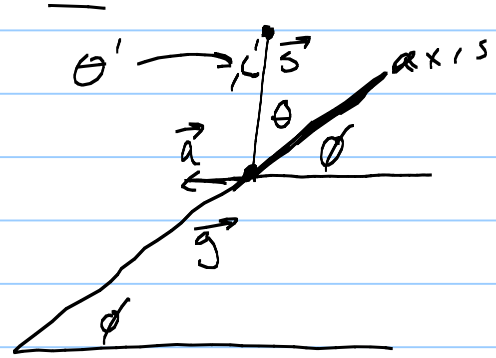
$$= \arcsin\left(\frac{\frac{1}{2}a}{g}\right)$$

$$= \arcsin\left(\frac{4\pi^2 R_e \cos 35^\circ}{2T^2 g}\right)$$

$$= \arcsin\left(\frac{4\pi^2 (6.37\times10^6)\cos 35^\circ}{2(8.64\times10^4)^2(9.8)}\right)$$

$$= \arcsin(0.141\times10^{-2})$$

$$= 0.0806^\circ$$



Another solution: use $\vec{g}$ (after deflection) as the vert. axis, and look at orthogonal components.

Here, $\theta \doteq$ deflection angle, assume very small, so $\phi \doteq 35^\circ$

$$\vec{a}_v = \vec{g}_v + \vec{S}_v \qquad \vec{a}_h = \vec{g}_h + \vec{S}_h$$

Since $\vec{S} + \vec{F}_g = \vec{F}_c$,

$\vec{S}$ = rope, $\vec{F}_c$ = centripetal

Note: $\vec{g}_h = 0$, $|\vec{g}_v| = g$

$$v = \frac{2\pi R_e \cos\phi}{T}, \quad |\vec{a}| = \frac{v^2}{R_e \cos\phi} = \frac{4\pi^2 R_e \cos\phi}{T^2}$$

$\therefore$ vert: $-\left(\frac{4\pi^2 R_e \cos\phi}{T^2}\right)\cos\phi = -g + \frac{S}{m}\cos\theta$

horiz: $\left(\frac{4\pi^2 R_e \cos\phi}{T^2}\right)\sin\phi = \frac{S}{m}\sin\theta$

To cancel S, divide horiz by vert.

$$\frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{4\pi^2 R_e \cos\phi}{T^2}\right)\sin\phi}{g - \left(\frac{4\pi^2 R_e \cos\phi}{T^2}\right)\cos\phi}$$

$$\tan\theta = \frac{4\pi^2 R_e \cos\phi \sin\phi}{gT^2 - 4\pi^2 R_e \cos^2\phi}$$

$$= \frac{4\pi^2(6.37\times10^6)(\cos 35^\circ)(\sin 35^\circ)}{(9.8)(8.64\times10^4)^2 - 4\pi^2(6.37\times10^6)(\cos 35^\circ)^2}$$

$$= 0.00162$$

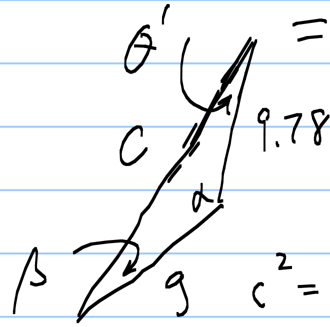
$$\therefore \theta = 0.0928^\circ$$

θ is a close approximation of real deflection angle. Real deflection angle is θ'. To calculate, determine $\frac{S}{m}$, then use law of sines.

$$\text{horiz: } \left(\frac{4\pi^2 R_e \cos\phi}{T^2}\right)\sin\phi = \frac{S}{m}\sin\theta$$

$$\frac{S}{m} = \frac{4\pi^2(6.37\times10^6)(\cos 35^\circ)(\sin 35^\circ)}{(8.64\times10^4)^2(\sin(0.0928^\circ))}$$

$$= 9.78 \text{ m/sec}^2$$



$$\alpha = 180^\circ - 0.0928^\circ$$
$$= 179.9072^\circ$$

$$c^2 = (9.8)^2 + (9.78)^2 - 2(9.8)(9.78)\cos\alpha$$

$$c = 19.577, \quad \frac{\sin\theta'}{9.8} = \frac{\sin 179.9072^\circ}{19.577},$$

$$\theta' = 0.0465^\circ$$

Could assume line from rope top to earth center is parallel to axis of $\vec{g}$, so that $\theta' = \theta$. But note that $\theta = \theta'$ + corner angle, as θ is an exterior angle of the triangle. Also, the triangle is virtually isosceles ($9.78 \approx 9.8$), so θ' is close to $\frac{1}{2}\theta$.

35. $v = v_t\left(1 - e^{-t/\tau}\right)$

(a) $v_t = \frac{mg}{b} = \frac{(3\times10^{-3})(9.8)}{b} = 2\times10^{-2}$, $b = 1.47\ \frac{kg}{sec}$

(b) $0.632\, v_t = v_t\left(1 - e^{-t/\tau}\right)$, $e^{-t/\tau} = 1 - 0.632$

$\ln(0.368) = -t/\tau$, $t = -\frac{m}{b}\ln(0.368) = 2.04\times10^{-3}$ sec

(c) At terminal speed, resistive force $= mg = bv_t$
$= (3\times10^{-3})(9.8) = 2.94\times10^{-2}$ N

41. (a) Terminal speed is when $F = 0$

$\therefore$ $Cv^2 = mg$, $v = \sqrt{\frac{mg}{C}} = \sqrt{\frac{(4.8\times10^{-4})(9.8)}{2.5\times10^{-5}}}$

$= 13.7$ m/sec

(b) Use spreadsheet

The time col is self evident

Value in position cell uses prior cell position value, multiplies prior velocity times 0.2

Velocity uses prior acc. times 0.2 sec, adds to prior veloc.

Acc. uses current velocity.

t	Position	Velocity	Acceleration
t'=t+0.20	x'=x+(t)v	v'=v+(t)a	a'=(0.0521)(v*v) - g
0.00	0.00	0.00	-9.80
0.20	0.00	-1.96	-9.60
0.40	-0.39	-3.88	-9.02
0.60	-1.17	-5.68	-8.12
0.80	-2.30	-7.31	-7.02
1.00	-3.77	-8.71	-5.85
1.20	-5.51	-9.88	-4.71
1.40	-7.48	-10.82	-3.70
1.60	-9.65	-11.56	-2.84
1.80	-11.96	-12.13	-2.14
2.00	-14.39	-12.56	-1.59
2.20	-16.90	-12.87	-1.17
2.40	-19.47	-13.11	-0.85
2.60	-22.09	-13.28	-0.62
2.80	-24.75	-13.40	-0.45
3.00	-27.43	-13.49	-0.32
3.20	-30.13	-13.55	-0.23
3.40	-32.84	-13.60	-0.17
3.60	-35.56	-13.63	-0.12
3.80	-38.28	-13.66	-0.08
4.00	-41.01	-13.67	-0.06
4.20	-43.75	-13.68	-0.04
4.40	-46.49	-13.69	-0.03
4.60	-49.23	-13.70	-0.02
4.80	-51.96	-13.70	-0.02
5.00	-54.71	-13.71	-0.01
5.20	-57.45	-13.71	-0.01
5.40	-60.19	-13.71	-0.01
5.60	-62.93	-13.71	0.00
5.80	-65.67	-13.71	0.00
6.00	-68.42	-13.71	0.00
6.20	-71.16	-13.71	0.00
6.40	-73.90	-13.71	0.00

44. (a). Use spreadsheet

Break up into components

Using $F = -bv = ma$, $a_x = -\frac{bv_x}{m}$,

$a_y = -9.8 - \frac{bv_y}{m}(\text{sign}(v_y))$. So, when object starts falling, $V_y < 0$, so resistive force opposes gravity.

$X_f = X_i + V_{x_i}(\Delta t)$

$Y_f = Y_i + V_{y_i}(\Delta t)$

$V_{x_f} = V_{x_i} + a_{x_i}(\Delta t)$

$V_{y_f} = V_{y_i} + a_{y_i}(\Delta t)$

T	X	Vx	Ax	Y	Vy	Ay	Vangle
0.00	0.00	81.92	-81.92	0.00	57.36	-67.16	35.00
0.10	8.19	73.72	-73.72	5.74	50.64	-60.44	34.49
0.20	15.56	66.35	-66.35	10.80	44.60	-54.40	33.91
0.30	22.20	59.72	-59.72	15.26	39.16	-48.96	33.25
0.40	28.17	53.74	-53.74	19.18	34.26	-44.06	32.52
0.50	33.55	48.37	-48.37	22.60	29.86	-39.66	31.68
0.60	38.38	43.53	-43.53	25.59	25.89	-35.69	30.74
0.70	42.74	39.18	-39.18	28.18	22.32	-32.12	29.67
0.80	46.65	35.26	-35.26	30.41	19.11	-28.91	28.45
0.90	50.18	31.74	-31.74	32.32	16.22	-26.02	27.07
1.00	53.35	28.56	-28.56	33.94	13.62	-23.42	25.49
1.10	56.21	25.71	-25.71	35.30	11.27	-21.07	23.68
1.20	58.78	23.14	-23.14	36.43	9.17	-18.97	21.62
1.30	61.09	20.82	-20.82	37.35	7.27	-17.07	19.25
1.40	63.18	18.74	-18.74	38.07	5.56	-15.36	16.54
1.50	65.05	16.87	-16.87	38.63	4.03	-13.83	13.43
1.60	66.74	15.18	-15.18	39.03	2.64	-12.44	9.88
1.70	68.25	13.66	-13.66	39.30	1.40	-11.20	5.85
1.80	69.62	12.30	-12.30	39.44	0.28	-10.08	1.30
1.90	70.85	11.07	-11.07	39.47	-0.73	-10.53	-3.76
2.00	71.96	9.96	-9.96	39.39	-1.78	-11.58	-10.14
2.10	72.95	8.96	-8.96	39.21	-2.94	-12.74	-18.15
2.20	73.85	8.07	-8.07	38.92	-4.21	-14.01	-27.58
2.30	74.66	7.26	-7.26	38.50	-5.61	-15.41	-37.71
2.40	75.38	6.53	-6.53	37.94	-7.16	-16.96	-47.60
2.50	76.03	5.88	-5.88	37.22	-8.85	-18.65	-56.40
2.60	76.62	5.29	-5.29	36.34	-10.72	-20.52	-63.72
2.70	77.15	4.76	-4.76	35.27	-12.77	-22.57	-69.54
2.80	77.63	4.29	-4.29	33.99	-15.02	-24.82	-74.07
2.90	78.06	3.86	-3.86	32.49	-17.51	-27.31	-77.57
3.00	78.44	3.47	-3.47	30.74	-20.24	-30.04	-80.26
3.10	78.79	3.13	-3.13	28.71	-23.24	-33.04	-82.34
3.20	79.10	2.81	-2.81	26.39	-26.55	-36.35	-83.95
3.30	79.38	2.53	-2.53	23.73	-30.18	-39.98	-85.21
3.40	79.64	2.28	-2.28	20.72	-34.18	-43.98	-86.19
3.50	79.86	2.05	-2.05	17.30	-38.58	-48.38	-86.96
3.60	80.07	1.85	-1.85	13.44	-43.41	-53.21	-87.57
3.70	80.25	1.66	-1.66	9.10	-48.73	-58.53	-88.05
3.80	80.42	1.49	-1.49	4.23	-54.59	-64.39	-88.43
3.90	80.57	1.35	-1.35	-1.23	-61.03	-70.83	-88.74

(b) Range (horizontal) occurs around $t = 3.9$ sec and is about 80.5 m

(c) From spreadsheet, maximum range occurs at $\approx 23°$!

55. 

$\uparrow N\cos\theta$

$\downarrow mg$

Note that $r = L\cos\theta$

$$mg = N\cos\theta$$

$$F_{centripetal} = ma = \frac{mv^2}{r} = N\sin\theta$$

$$\therefore N = \frac{mg}{\cos\theta}, \quad \frac{mv^2}{L\cos\theta} = \frac{mg\sin\theta}{\cos\theta},$$

$$\therefore v^2 = Lg\sin\theta, \quad v = \sqrt{Lg\sin\theta}$$

58. For penny, centripetal force $= \frac{mv^2}{r} = \mu_{sp} mg$,

so $V_{max}^2 = \mu_{sp} g r$, $V_{max} = \sqrt{\mu_{sp} g r}$

Assuming penny sticks to block forever,

$$V_{max} = \sqrt{\mu_{sb}\, g r}$$

Since $\mu_{s\,penny} = 0.450 < 0.750 = \mu_{s\,block}$,

penny slides first at $v = \sqrt{(0.45)(9.8)(0.012)}$

$= 0.0529$ m/sec

$2\pi r = 2\pi(0.012) = 0.0754$ m / revolution

$\therefore \dfrac{.0529}{.0754} \times 60 \dfrac{sec}{min} = 42.1$ rpm

64. (a). $R = \dfrac{V_i^2 \sin 2\theta_i}{g}$, $V_i = \sqrt{\dfrac{Rg}{\sin 2\theta}}$

$= \sqrt{\dfrac{(285)(9.8)}{\sin 96°}}$

$= 53.0$ m/sec.

$V_f = V_i + at$, $-53.0 = 53.0 - 9.8(t)$

$t = 10.8$ secs.

(b) $\dfrac{2\pi R_e \cos 35°}{86400 \text{ secs}} = \dfrac{2\pi(6.37 \times 10^6 \text{ m})(\cos 35°)}{86400 \text{ secs}}$

$= 381$ m/sec.

(c) $R_e \theta = 285$ m (θ in radians)

$\therefore \theta = \dfrac{285}{6.37 \times 10^6} \times \dfrac{360 \text{ degrees}}{2\pi \text{ radians}} = 2.56 \times 10^{-3}$ degrees

$\therefore \dfrac{2\pi R_e \cos 35^\circ}{T} - \dfrac{2\pi R_e \cos(35^\circ - \Delta)}{T}$

$= \dfrac{2\pi (6.37 \times 10^6)}{86400} \left(\cos 35^\circ - \cos(35^\circ - \Delta)\right)$

$= 0.0119$ m/sec

(d). In flight for 10.8 sec. $\therefore (0.0119)(10.8) = 0.129$ m