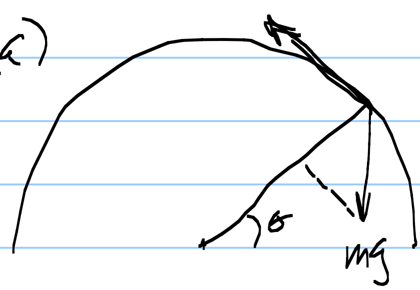
Chapter 7 - Work and Kinetic Energy

Note Title 2/7/2005

5. (a) $(16.0)(\cos 25^\circ)(2.20) = 31.9\ J$
   (b) 0 for normal force
   (c) 0 for gravity
   (d) same as (a)

14. $|F||v| \cos\theta = \vec{F} \cdot \vec{v}$
   For $\vec{F}$, $118^\circ - 90^\circ = 28^\circ$, for $\vec{v}$, $132^\circ - 90^\circ = 42^\circ$
   $\therefore\ 90 - (28 + 42) = 20^\circ$
   $\therefore (32.8)(0.173) \cos 20^\circ = 5.33$ Watts

25. (a)



The component of gravity along the dotted line ($\perp$ to radius R) is $g \sin(90 - \theta) = g \cos\theta$

$\therefore$ Force of gravity $= -mg\cos\theta$

$\therefore$ To oppose this force exactly for constant speed, F of rope $= mg\cos\theta$

(b) Work from 0° to 90°
A small displacement is $ds = R\,d\theta$ (θ in radians).

$$\therefore \int_0^{\frac{\pi}{4}} \vec{F}\cdot d\vec{s} = \int_0^{\frac{\pi}{4}} (mg\cos\theta)(R)\,d\theta$$

$$= mgR \int_0^{\frac{\pi}{4}} \cos\theta\, d\theta = mgR\sin\theta \Big|_0^{\frac{\pi}{4}} = mgR$$

27. (a) $\frac{1}{2}mv^2 = \frac{1}{2}(0.600)(2.00)^2 = 1.20\text{ J}$

(b) $\frac{1}{2}(0.600)v^2 = 7.50$, $\frac{15}{0.6} = 25 = v^2$, $v = 5.00\text{ m/sec.}$

(c) $7.50 - 1.20 = 6.30\text{ J}$

37. Force of frictions slows & stops sled.

$F_f = \mu mg = (0.1)mg$

Initial K.E. $= \frac{1}{2}mv^2 = \frac{1}{2}m(20)^2 = 2m$

Final K.E. $= 0$

$\therefore 2m = 0.1mg\,d$, $d = \frac{20}{g} = 2.04\text{ m}$

40. The tension in the rope of Atwood machine is:

$$T = \frac{2m_1 m_2}{m_1 + m_2}\,g$$

Look at $m_2 > m_1$.

Force of gravity on $m_2 = m_2 g$

$\therefore$ For m_2, $(F_g - T)d$ = work done on m_2

$$\therefore \left(m_2 - \frac{2m_1 m_2}{m_1 + m_2}\right) g\ (0.4\ m)$$

$$= \left(0.3 - \frac{2(0.3)(0.2)}{0.3+0.2}\right)(9.8)(0.4) = 0.235\ J$$

$$\therefore \frac{1}{2} m_2 v^2 = \frac{1}{2}(0.3)\,v^2 = 0.235,\ v^2 = \frac{2(0.235)}{0.3}$$

$V = 1.25$ m/sec

47. (a) Must find force of motor, since gravity also working.

Net acceleration: $V_f = at$, $1.75 = a(3.0)$,
$a = 0.55\ m/sec^2$
$\therefore$ Force by motor $= m(0.55 + g)$
$= (650)(0.55 + 9.8) = 6727.5\ N$

$$V_f^2 = 2ad,\quad d = \frac{(1.75)^2}{2(0.55)} = 2.78\ m$$

$$\therefore \frac{F \cdot d}{\Delta t} = \frac{(6727.5)(2.78)}{(3.0)} = 6243\ \text{Watts}$$

(b) At cruising speed, $F = mg$, $\therefore$ power is $mgv = (650)(9.8)(1.75) = 11,148\ W$

53. (a). $K.E. = mc^2\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right)$

$\therefore$ 0.5c to 0.75c,

$$\Delta KE = mc^2\left(\frac{1}{\sqrt{1-(0.75)^2}} - \frac{1}{\sqrt{1-(0.5)^2}}\right)$$

$$= mc^2(1.51 - 1.15) = mc^2(0.357)$$

$$= (1.67\times10^{-27})(3\times10^8)^2(0.357)$$

$$= 5.37\times10^{-11}\ \text{Joule}$$

(b) $\Delta KE = mc^2\left(\frac{1}{\sqrt{1-(0.995)^2}} - \frac{1}{\sqrt{1-(0.5)^2}}\right)$

$$= mc^2(10.0 - 1.15) = (8.86)\,mc^2$$

$$= (1.67\times10^{-27})(3\times10^8)^2(8.86)$$

$$= 1.33\times10^{-9}\ \text{Joule}$$

59. (a) $x(t) = t + 2.0t^3$

$\therefore v(t) = x'(t) = 1 + 6.0t^2$

$\therefore \frac{1}{2} m v(t)^2 = \frac{1}{2}(4)(1+6.0t^2)^2$

$= 2(1 + 12t^2 + 36t^4)$

$= (72t^4 + 24t^2 + 2)$ Joules

(b) $a(t) = v'(t) = 12.0t$ m/sec^2

$F(t) = ma(t) = 48.0t$ Newtons

(c) $P(t) = \frac{d\,KE(t)}{dt} = (288t^3 + 48t)$ Watts

(d) $t=0$: $KE = 2.0$ J
$t=2$; $KE = 72(2)^4 + 24(2)^2 + 2 = 1250$ J

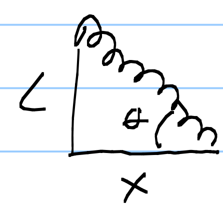
$\therefore 1250 - 2 = 1248 = 1.25 \times 10^3$ J

66. (a) The length of each spring, when the particle is displaced by x, is $\sqrt{x^2+L^2}$

∴ The displacement for each spring from equilibrium is: $\sqrt{x^2+L^2} - L$

∴ The force of each spring on the particle is: $-K\left(\sqrt{x^2+L^2} - L\right)$

The force component along the x axis is



$F\cos\theta$, and $\cos\theta$ is $\dfrac{x}{\sqrt{x^2+L^2}}$

∴ Each spring contributes:

$$-K\left(\sqrt{x^2+L^2} - L\right)\left(\frac{x}{\sqrt{x^2+L^2}}\right)$$

$$= -Kx\left(1 - \frac{L}{\sqrt{x^2+L^2}}\right)$$

Since there are 2 springs,

$$F(x) = -2Kx\left(1 - \frac{L}{\sqrt{x^2+L^2}}\right)$$

(b) $$\int_A^0 F(x)\,dx = \int_A^0 -2Kx\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)dx$$

$$= -\int_A^0 2Kx\,dx + \int_A^0 \frac{2KLx}{\sqrt{x^2+L^2}}\,dx$$

$$= -Kx^2\Big|_A^0 + 2KL\sqrt{x^2+L^2}\Big|_A^0$$

$$= KA^2 + 2KL^2 - 2KL\sqrt{A^2+L^2}$$

$$= K\left(A^2+2L^2 - L\sqrt{A^2+L^2}\right)$$

This quantity is always positive:

Let $A^2+L^2=C^2$ (right triangle)

$\therefore$ Quantity in parenthesis becomes:

$$C^2+L^2-LC = C^2 + L(L-C)$$
$$= C^2 - L(C-L)$$

Note $L<C$ and $C-L \le C$. $\therefore L(C-L) \le C^2$

$\therefore C^2 - L(C-L) \ge 0$