Chapter 8 - Potential Energy and Conservation of Energy

Note Title 2/25/2005

4. (a). Given constant force, which doesn't vary over time or position, or object velocity. Show that force is conservative.

Let $\vec{F}(x,y,z) = a\hat{i} + b\hat{j} + c\hat{j}$

Let $\vec{r}(t) = (x(t), y(t), z(t))$

$\therefore \frac{d\vec{r}(t)}{dt} = (x'(t), y'(t), z'(t))$

$$\therefore W = \int_{t=i}^{t=f} \vec{F} \cdot d\vec{r} = \int_{t=i}^{t=f} (a,b,c)\cdot(x'(t), y'(t), z'(t))\,dt$$

$$= a\,x(t) + b\,y(t) + c\,z(t) \Big|_i^f$$

$$= \vec{F} \cdot (\vec{r}(f) - \vec{r}(i))$$

$\therefore$ Work only depends on beginning & ending position, and so is path independent.
$\therefore \vec{F}$ is conservative.

(b). OAC: $\int_{x=0,y=0}^{x=5,y=0} \vec{F}\cdot d\vec{x} + \int_{y=0,x=5}^{y=5,x=5} \vec{F}\cdot d\vec{y}$

$$= \int_0^5 (3,4)\cdot(1,0)\,dx + \int_0^5 (3,4)\cdot(0,1)\,dy$$

$$= 3x\Big|_0^5 + 4y\Big|_0^5 = 15 + 20 = 35\text{ J}$$

OBC: $$\int_{x=0,y=0}^{x=0,y=5} \vec{F}\cdot\vec{dy} + \int_{x=0,y=5}^{x=5,y=5} \vec{F}\cdot\vec{dx}$$

$$= 20 + 15 = 35\text{ J}$$

OC: $$\int_{x=0,y=0}^{x=5,y=5} \vec{F}\cdot\vec{dr} = \int_{0,0}^{5,5} (3,4)\cdot(1,1) = \int_{0,0}^{5,5} 3dx + 4dy$$

$$= 3x\Big|_0^5 + 4y\Big|_0^5 = 15 + 20 = 35\text{ J}$$

10. (a). $\Delta U = (0.5\text{ Kg})(9.8\text{ m/sec}^2)(20\text{ m})$

$KE_{peak} = \frac{1}{2} m v_h^2$

$KE_{start} = \frac{1}{2} m v_i^2 = \frac{1}{2} m (v_h^2 + v_v^2)$

$\therefore \frac{1}{2} m (v_h^2 + v_v^2) = m(9.8)(20.0) + \frac{1}{2} m v_h^2$

$\therefore v_v^2 = 2(9.8)(20.0)$

$\therefore v_v = \sqrt{2(9.8)(20)} = 19.8\text{ m/sec} = \vec{v}_{i_{vertical}}$

(b). $W_g = -\Delta U = -(0 - mg\,(60m))$

$= (0.5\,kg)(9.8)(60) = 294\,J$

(c) $\Delta KE + \Delta U = 0$, since gravity is conservative.

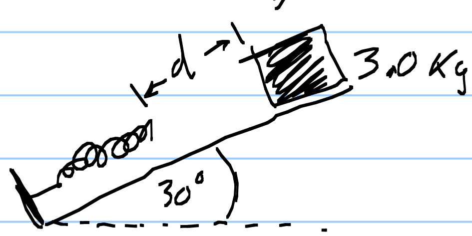
$\therefore KE_p + 294 = KE_B$

$\frac{1}{2}m(V_{v_i}^2 + V_h^2) + 294 = \frac{1}{2}m(V_{v_f}^2 + V_h^2)$

V_h unchanged, since gravity does no work on horizontal velocity. Note $V_{v_i}^2 = 2(9.8)(20)$ from (a).

$\therefore \frac{1}{2}(0.5)(9.8)(20) + 294 = \frac{1}{2}(0.5)(V_{v_f}^2)$

$\therefore V_{v_f} = 39.6\ m/sec \qquad V_h = 30.0\ m/sec.$

11. 

$(d + 0.2m)\sin 30° = \Delta h$

$\Delta U = mg\Delta h$

$\therefore \Delta U = \frac{1}{2}(d + 0.2)\,mg$

Change in spring P.E. $= \frac{1}{2}kx^2 = \frac{1}{2}(400\,n/m)(0.2m)^2$

$= (200)(0.04)$

$= 8\,J$

$\therefore\ 8 = \frac{1}{2}(d + 0.2)(3.0)(9.8)$, $d = 0.344$ meter

42. $U(x,y) = 3x^3y - 7x$

$$F(x,y) = \left(-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}\right) = \left(-9x^2y + 7, -3x^3\right)$$

47. $U_f(x) = -\int_{x_i}^{x_f} F_x\,dx + U_i$, here $U_i = 0$ at $x = 0$ (spring not stretched)

From prob. 66, chp. 7, $F_x = -2Kx\left(1 - \frac{L}{\sqrt{x^2+L^2}}\right)$

$$U(x) = -\int_0^x \left(-2Kx + \frac{2KxL}{\sqrt{x^2+L^2}}\right) dx$$

$$= -\int_0^x -2Kx\,dx - \int_0^x \frac{KL(2x)}{\sqrt{x^2+L^2}}\,dx$$

$$= Kx^2\Big|_0^x - 2KL\sqrt{x^2+L^2}\Big|_0^x$$

$$= Kx^2 - 2KL\sqrt{x^2+L^2} + 2KL^2$$

$$= Kx^2 + 2KL\left(L - \sqrt{x^2+L^2}\right)$$

Note: Let $x^2 + L^2 = c^2$ (right triangle)

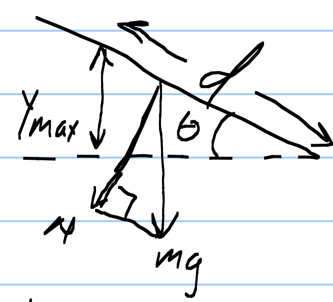
$$U(x) = k[x^2 + 2L^2 - 2Lc]$$

$$= k[c^2 + L^2 - 2Lc] = k[c-L]^2$$

$c > L$, so, $U(x) > 0$, for all x

50. $mgh = mgy_{max}$ + amount lost to friction

Friction lost $= F_f \cdot d$, $d = \dfrac{y_{max}}{\sin\theta}$



$F_f = \mu_k N = \mu_k mg\cos\theta$

$N = mg\cos\theta$

$\therefore$ friction loss $= (\mu_k mg\cos\theta)\left(\dfrac{y_{max}}{\sin\theta}\right)$

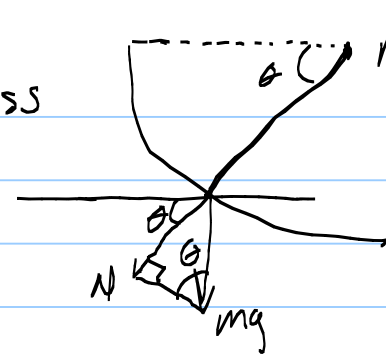
$$= \mu_k mg y_{max} \cot\theta$$

$\therefore mgh = mgy_{max} + \mu_k mg y_{max} \cot\theta$

$$h = y_{max} + \mu_k y_{max} \cot\theta$$

$$\therefore y_{max} = \frac{h}{1 + \mu_k \cot\theta}$$

53. (c) $F_f \cdot d$ = friction loss



$\therefore N = m\vec{g}\sin\theta$

$$\mu_k \int_0^{\frac{\pi}{2}} mg\sin\theta \cdot r\,d\theta$$

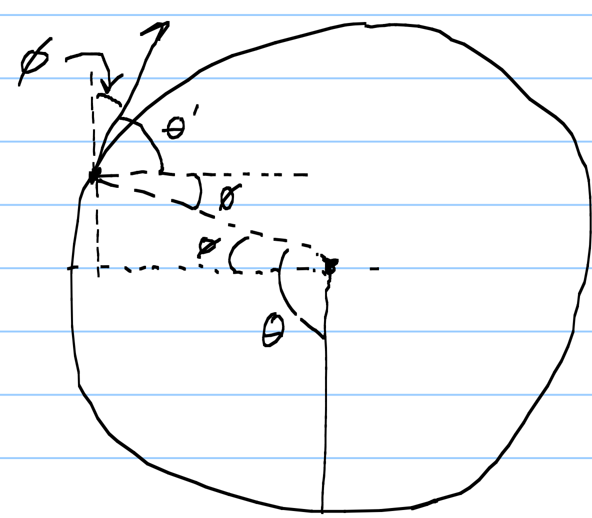
= energy of friction loss

$$\therefore \mu_k mgr \int_0^{\frac{\pi}{2}} \sin\theta\,d\theta \qquad -\cos\theta\Big|_0^{\frac{\pi}{2}} = 1$$

$$\therefore \mu_k mgr = \text{friction loss energy}$$
$$= PE - KE$$
$$= mgr - \tfrac{1}{2}mv_B^2$$

$$\therefore \mu_k = \frac{2gr - v_B^2}{2gr}$$

68.



Energy at top of circle $= mg(2R) + \frac{1}{2}mRg$

String always perpendicular so doesn't add or subtract energy at any point.

$\theta' = 90 - \phi \qquad \therefore \; \theta + \theta' = 180$

$\theta = 90 + \phi$

At point of release, $x = R\cos\phi$

$y = R\sin\phi$

From Eq. 4.12 for parabolic motion (p. 82),

$$y = (\tan\theta')x - \left(\frac{g}{2v_i^2\cos^2\theta'}\right)x^2 \qquad [1]$$

At release, $x = R\cos\phi = R\cos(90-\theta') = R\sin\theta'$

$y = R\sin\phi = R\sin(90-\theta') = R\cos\theta'$

From energy, at point of release,

$$\tfrac{1}{2}mv_i^2 + \left[(R\sin\phi)mg + mgR\right] = mg2R + \tfrac{1}{2}mRg$$

$$\therefore \tfrac{1}{2}v_i^2 + R\cos\theta' g + gR = g2R + \tfrac{1}{2}Rg$$

$$v_1^2 + 2R\cos\theta' g = 2gR + Rg$$

$$v_i^2 = 3gR - 2gR\cos\theta'$$

$$v_i^2 = gR(3 - 2\cos\theta')$$

From [1],

$$-R\cos\theta' = (\tan\theta')(R\sin\theta') - \frac{g \cdot R^2\sin^2\theta'}{2gR(3-2\cos\theta')\cos^2\theta'}$$

$$0 = \tan\theta'\sin\theta' + \cos\theta' - \frac{\sin^2\theta'}{2(3-2\cos\theta')\cos^2\theta'}$$

$$0 = \frac{\sin^2\theta'}{\cos\theta'} + \cos\theta' - \frac{\sin^2\theta'}{2(3-2\cos\theta')\cos^2\theta'}$$

$$0 = \sin^2\theta'[2(3-2\cos\theta')]\cos\theta' + \cos^3\theta'[2(3-2\cos\theta')] - \sin^2\theta'$$

From spreadsheet, $\theta' \approx 79.5^\circ$

$\therefore\ \theta = 180 - 79.5^\circ = 100.5^\circ$