Chapter 9 - Linear Momentum and Collisions

Note Title 4/11/2005

6. (a). $(3M)(2.0\ m/sec) = (m)v$, $v = 6.0\ m/sec$

(b). $\frac{1}{2}(3m)(2.0)^2 + \frac{1}{2}(m)(6.0)^2 = 24M$

Energy of spring $= 24M$ since energy is conserved.

15. $(200)(50.0) = (200)(40.0) + (46g)(v)$

$$\frac{(15\ m/sec)(200\ g)}{46\ g} = 65.2\ m/sec.$$

18. Pendulum bob gets a velocity of $\frac{mv}{2m}$ since bullet loses $\frac{mv}{2}$ momentum.

To go to the peak, the plumb bob must must convert kinetic energy to surmount the change in potential energy: $2Mlg$

$\therefore$ K.E. of bob $= \frac{1}{2}m\left(\frac{mv}{2m}\right)^2 = \frac{m^2v^2}{8m}$

$\therefore \frac{m^2v^2}{8m} = 2Mlg$, $v^2 = \frac{16M^2lg}{m^2}$

$\therefore v = \frac{4M}{m}\sqrt{lg}$ is minimum speed of bullet.

25. (a). From elastic head-on equations, with particle 2 at rest:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i}$$

$$v_{2f} = \left(\frac{2 m_1}{m_1 + m_2}\right) v_{1i}$$

$\therefore$ For neutron (#1) and carbon (#2) nucleus, $m_1 = 1$, $m_2 = 12$.

$$\therefore v_{1f} = -\frac{11}{13} v_{1i}$$

$$v_{2f} = \frac{2}{13} v_{1i}$$

Initial K.E. neutron: $\frac{1}{2} m_1 v_{1i}^2$

Final carbon K.E.: $\frac{1}{2} 12 m_1 \left(\frac{2}{13} v_{1i}\right)^2 = \frac{48 m_1 v_{1i}^2}{338}$

$\therefore$ Fraction transferred $= \frac{96}{338} = 0.284$

(b) Carbon: $(1.6 \times 10^{-13}\ \text{J})(0.284) = 4.54 \times 10^{-14}\ \text{J}$

Neutron: $1.6 - 0.454 = 1.14 \times 10^{-13}\ \text{J}$

26. Initial P.E. m_1 : $5 m_1 g$ = K.E. = $\frac{1}{2} m_1 V_1^2$,

so $V_1^2 = 10g$, $V_1 = \sqrt{10g}$

Since m_2 at rest, recoil velocity of m_1 is:

$$V_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_{1i} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \sqrt{10g} \quad (V_{1f} < 0)$$

$$\therefore \text{Recoil K.E.} = \frac{1}{2} m_1 V_{1f}^2 = \frac{1}{2} m_1 (10g) \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

$$\therefore \text{Height} = m_1 g h = 5 m_1 g \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2,$$

$$\text{so } h = 5 \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 = 5 \left(\frac{5 - 10}{5 + 10}\right)^2$$

$$= \frac{5}{9} \text{ meters}$$

27. $\frac{1}{2} m V_c^2 = F_f d = \mu N d = (0.650)(mg)(7.50)$

$$\therefore V_c^2 = 2g(0.650)(7.50), \quad V_c = \sqrt{2g(0.650)(7.50)}$$

Where V_c = Velocity (combined: bullet + block)

From conservation of momentum,

$(0.012) V_{bullet} = (0.012 + 0.100) V_c$,

$$\therefore V_{bullet} = \frac{112}{12} V_c = \frac{112}{12}\sqrt{2(9.8)(0.650)(7.50)}$$

$$= 91.2 \text{ m/sec.}$$

29. (a) Horiz. & vert. momentum conserved.

Horiz: $(90)(5.0) = (90+95)V_x$, $V_x = 2.43$ m/sec.

Vert: $(95)(3.0) = (90+95)V_y$, $V_y = 1.54$ m/sec.

$$\sqrt{V_x^2 + V_y^2} = \sqrt{2.48^2 + 1.54^2} = 2.88 \text{ m/sec}$$

$$\tan\theta = \frac{V_y}{V_x} = \frac{1.54}{2.48}, \quad \theta = 32.4^\circ$$

$\therefore$ 2.88 m/sec at 32.4° north of east.

(b) Energy Before: $\frac{1}{2}(90)(5.0)^2 + \frac{1}{2}(95)(3.0)^2$
$= 1.55 \times 10^3$ J

Energy After: $\frac{1}{2}(90+95)(2.88)^2 = 767$ J

$\therefore 1.55 \times 10^3 - 767 = 783$ J, lost to internal energy.

40. $$\frac{(2)(3) + (3)(2.5) + (2.5)(0) + (4)(-0.5)}{2+3+2.5+4} =$$

$$\frac{6 + 7.5 + 0 + (-2)}{11.5} = \frac{11.5}{11.5} = 1.0.$$ Center of mass at +1.0 (y-axis)

42. $\dfrac{(m_e)(0)+(m_m)(3.84\times10^8)}{m_e+m_m} = \dfrac{(7.36\times10^{22})(3.84\times10^8)}{(5.98+0.07)\times10^{24}}$

$= 4.67\times10^6$ m

CM $= 4.67\times10^6$ m from center of earth.

43. Horizontal coord: $\dfrac{2M_H(0.1)(\cos 53^\circ)}{2M_H+M_o} = \dfrac{2(0.602)(.1)}{2+16}$

$= 0.007$ nm from center of oxygen.

By symmetry, y coords. cancel.

45. (a). Rod of 30.0 cm long

$\lambda = 50.0\ g/m + 20.0x\ g/m^2$

$$\text{Mass of rod} = \int_0^{0.3} \lambda(x)\,dx$$

$$= \int_0^{0.3} (0.05 + 0.02x)\,dx = 0.05x + 0.01x^2\Big|_0^{0.3}$$

$= (0.05)(0.3) + (0.01)(0.09) = 0.015 + 0.0009$

$= .0159$ kg $= 15.9$ g

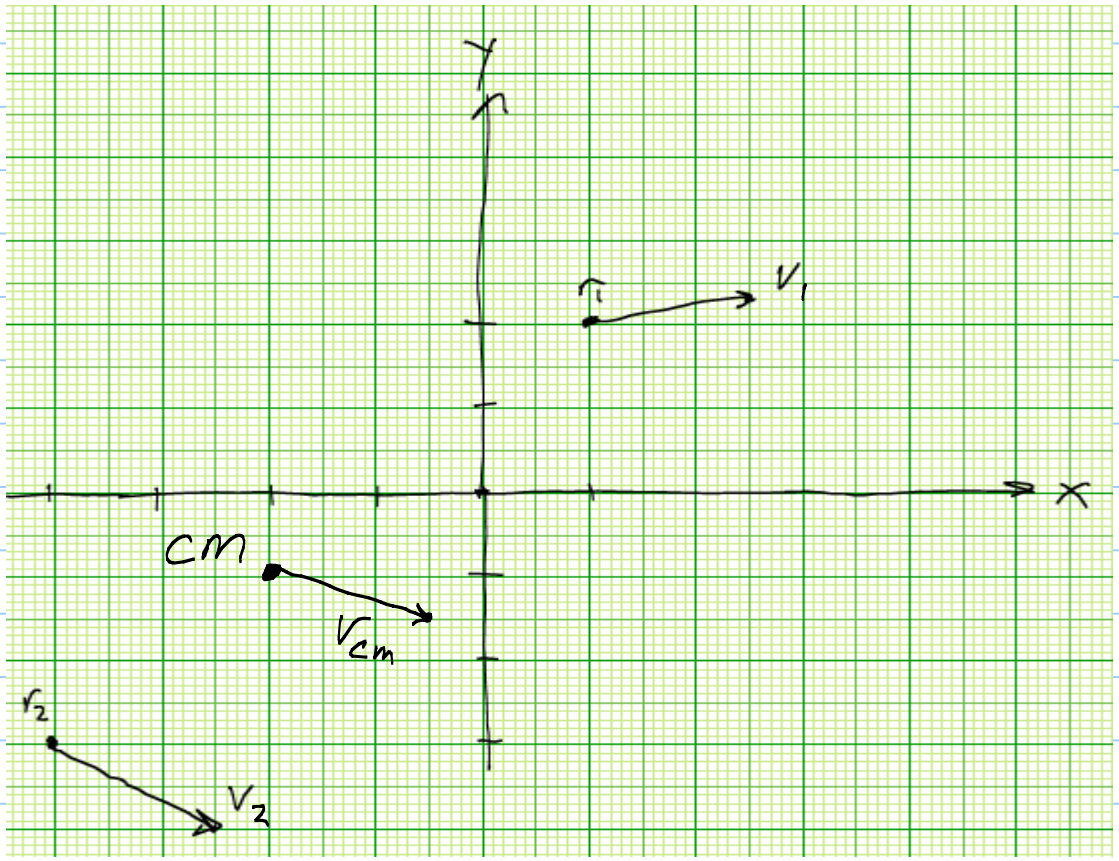
(b). $CM = \dfrac{1}{m}\displaystyle\int_0^{0.3} x\,\lambda(x)\,dx = \frac{1}{m}\int_0^{0.3} (0.05x + 0.02x^2)\,dx$

$$= \frac{1}{.0159}\left[0.025x^2 + \frac{0.02x^3}{3}\right]\Bigg|_0^{0.3}$$

$$= \frac{1}{0.0159}\left((0.025)(0.3)^2 + \frac{(0.02)(0.3)^3}{3}\right)$$

$$= 0.153\ m = 15.3\ cm \text{ from end.}$$

46. (a)



(b). $m_1 = 2.0\ kg$

$m_2 = 3.0\ kg$

$$CM = \frac{1}{m}\left(m_1\vec{r}_1 + m_2\vec{r}_2\right)$$

$$= \frac{1}{5}\left(2\cdot(1,2) + 3(-4,-3)\right)$$

$$= \frac{1}{5}\left((2,4) + (-12,-9)\right)$$

$$= \frac{1}{5}(-10,-5) = (-2,-1)$$

(c). $V_{cm} = \frac{1}{m}\left(m_1\vec{v}_1 + m_2\vec{v}_2\right)$

$$= \frac{1}{5}\left(2(3,0.5) + 3(3,-2)\right)$$

$$= \frac{1}{5}\left((6,1) + (9,-6)\right) = \frac{1}{5}(15,-5)$$

$$= (3,-1)$$

(d) Momentum $= m_1\vec{v}_1 + m_2\vec{v}_2$

$$= 2(3,0.5) + 3(3,-2)$$
$$= (6,1) + (9,-6)$$
$$= (15,-5) \text{ kg·m/sec}$$

This is the same as $M \cdot V_{cm} = 5(3,-1)$

47. No external forces, so center of mass is unchanged (assuming at rest at start).

Don't really know where Romeo & Juliet are positioned in boat. Assume boat

of uniform density.

Let R = distance Romeo to shore,
B = distance center of boat to shore.

$$\therefore CM_i = \frac{77(R) + 55(R-2.7) + 80(B)}{77+55+80}$$

After Juliet moves, Romeo & boat move Δx,
Juliet moves so that she is now at $R-\Delta x$

$$\therefore CM_f = \frac{77(R-\Delta x) + 55(R-\Delta x) + 80(B-\Delta x)}{77+55+80}$$

Since $CM_i = CM_f$,

$$77(R) + 55(R-2.7) + 80(B) = 77(R-\Delta x) + 55(R-\Delta x) + 80(B-\Delta x)$$

$$-55(2.7) = (77+55+80)(-\Delta x)$$

$$\therefore \Delta x = 0.700 \text{ meter}$$

51. (a) $\frac{dM}{dt} = 1.5\times10^4 \text{ kg/sec}$, $V_e = 2.6\times10^3 \text{ m/sec}$.

$$\therefore \text{Thrust} = (1.5\times10^4)(2.6\times10^3) = 3.9\times10^7 \text{ N}$$

(b). At the instant of take off, $F = ma$,

$F = \text{Thrust} - \text{gravity} = 3.9 \times 10^7 - (3 \times 10^6)(9.8)$

$= 9.6 \times 10^6$ N

$m = 3.0 \times 10^6$,

$\therefore a = \frac{9.6 \times 10^6}{3.0 \times 10^6} = 3.2$ m/sec^2

53. (a). From $V_f - V_i = V_e \ln\left(\frac{m_i}{m_f}\right)$, $V_i = 0$, $V_e = 2 \times 10^3$ m/sec

$m_f = 3.0 \times 10^3$ kg $\qquad V_f = 10 \times 10^4$ m/sec.

$\therefore 1.0 \times 10^4 = 2 \times 10^3 \ln\left(\frac{3 \times 10^3 + x}{3 \times 10^3}\right)$

$e^5 = \frac{3 \times 10^3 + x}{3 \times 10^3}$

$(148.4)(3 \times 10^3) = 3 \times 10^3 + x$,

$x = 4.42 \times 10^5$ kg fuel = 442 metric tons

(b). If $V_e = 5 \times 10^3$, Then

$1.0 \times 10^4 = 5 \times 10^3 \ln\left(\frac{3 \times 10^3 + x}{3 \times 10^3}\right)$,

$e^2 = \frac{3 \times 10^3 + x}{3 \times 10^3}$, $\quad x = 19.2$ metric tons

55. (a). Momentum of person: $60(4) = 240$ kg-m/sec.

Momentum of person + cart

$(60 + 120)V = 240$, $V = \frac{240}{180} = 1.33$ m/sec.

(b). Frictional force $= -\mu N = -\mu mg$

$= -(0.4)(60)(9.8) = -235.2$ Newtons

(c) $V_f - V_i = a\Delta t$, $a = F_f/m = -\frac{(0.4)(60)(9.8)}{60}$

$V_f = 1.33$, $V_i = 4.0$.

i.e., uniform deceleration relative to ground

$\therefore \Delta t = \frac{V_f - V_i}{a} = \frac{1.33 - 4.0}{-(0.4)(9.8)} = 0.68$ sec

(d) $\Delta p_{person} = m(V_f - V_i) = 60(1.33 - 4.0) = -160$

$\Delta p_{cart} = 120(1.33 - 0) = 160$ kg-m/sec.

(e) $V_f^2 - V_i^2 = 2ad$, $d = \frac{(1.33)^2 - (4.0)^2}{-(2)(0.4)(9.8)} = 1.81$ m

(f) $V_f^2 - V_i^2 = 2ad$, $a = -F_f/m_{cart} = \frac{(0.4)(60)(9.8)}{120}$

$\therefore d = \frac{(1.33)^2 - 0}{2\left[\frac{(0.4)(60)(9.8)}{120}\right]} = 0.454$ m

(g) $\Delta KE_{person} = \frac{1}{2} m \left(1.33^2 - 4.0^2\right)$

$= \frac{1}{2}(60)(1.33^2 - 4.0^2) = -427\ J$

(h) $\Delta KE_{cart} = \frac{1}{2} m v^2 = \frac{1}{2}(120)(1.33^2) = 107\ J$

(i) Rest of energy goes internal - heat between person (shoes) and cart.
Perfectly inelastic collision.

C5. Approach this from an impulse ($F \cdot \Delta t$) perspective, and assume free fall for an small mass $= \Delta m$ of the chain. Chain of uniform density $= m/L$. Total time of fall is: $x = \frac{1}{2} g T^2$, so $T = \sqrt{2x/g}$

So, look at $\Delta p = F(t)\,\Delta t$ at time t, and take limit as $\Delta t \to 0$.

At time t, chain dropping at speed of gt since started at rest.
Momentum of a small Δm is $\therefore \Delta m(gt)$
Over a small amount of time, the small Δm falls a distance of $(\text{speed})(\Delta T) = (gt)\Delta t$
$\therefore \Delta m = \frac{m}{L}(gt)\Delta t$

$\therefore$ momentum at time t is $\left[\frac{m}{L}(gt)(\Delta t)\right](gt)$

Since final momentum of packet Δm is 0,

$$\Delta p = \frac{mg^2t^2}{L}\Delta t$$

$$F(t) = \frac{\Delta p}{\Delta t} = \frac{mg^2t^2}{L}, \text{ and at time } t = T,$$

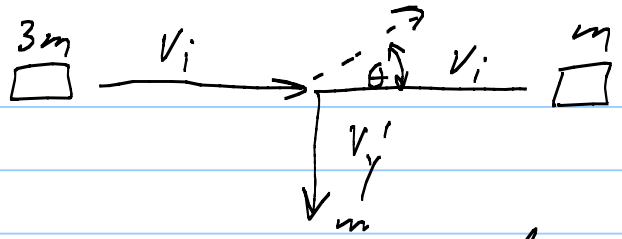
$$T^2 = (T2\pi/g)^2 = 2x/g$$

$$\therefore F = \frac{mg^2 2x}{L\,g} = \frac{2mgx}{L}$$

Now for the chain already at rest on the table, it's mass is $\frac{m}{L}x$, and so it's weight is $\frac{mgx}{L}$.

$\therefore$ Total force = Impulse force of chain coming down + weight of chain already on table

$$= \frac{2mgx}{L} + \frac{mgx}{L} = \frac{3mgx}{L}$$

73. 

(a) Find final speeds of the two masses.

Momentum before collision:

$$3mv_i\hat{i} - mv_i\hat{i} = 2mv_i\hat{i}$$

Momentum after collision:

for m: $-mv_y'\hat{j}$

for 3m: $+mv_y'\hat{j} + 2mv_i\hat{i}$ in order to conserve momentum

Let v_y = vertical 3m speed after crash

v_x = horiz. 3m speed after crash

$\therefore 3mv_y = mv_y'$, $3mv_x = 2mv_i$

$\therefore v_x = \frac{2}{3}v_i$, $v_y = \frac{1}{3}v_y'$

Since collision elastic, KE conserved.

$\therefore$ Before: $3mv_i^2 + mv_i^2$

After: $mv_y'^2 + 3m\left[\left(\frac{2}{3}v_i\right)^2 + \left(\frac{1}{3}v_y'\right)^2\right]$

$$\therefore\ 4v_i^2 = v_y'^2 + 3\left[\frac{4}{9}v_i^2 + \frac{1}{9}v_y'^2\right]$$

$$= v_y'^2 + \frac{4}{3}v_i^2 + \frac{1}{3}v_y'^2$$

$$\therefore\ \frac{8}{3}v_i^2 = \frac{4}{3}v_y'^2,\quad 2v_i^2 = v_y'^2,$$

$$v_y' = \sqrt{2}\,v_i$$

$\therefore$ Final speed for m: $\sqrt{2}\,v_i$

for 3m: $\sqrt{\left(\frac{2}{3}v_i\right)^2 + \left(\frac{\sqrt{2}}{3}v_i\right)^2}$

$$= \sqrt{\frac{6}{9}v_i^2} = \sqrt{2/3}\,v_i$$

(6) Angle of θ:

$$\tan\theta = \frac{v_y}{v_x} = \frac{\frac{\sqrt{2}}{3}v_i}{\frac{2}{3}v_i} = \frac{\sqrt{2}}{2}$$

$$\therefore\ \theta = 35.3^\circ$$

75. From spread sheet, $v_f = 3.72$ km/sec

$d = 153$ km

	A	B	C	D	E
1	360.00	Rocket Mass			
2	1,500.00	Exhaust velocity			
3	2.50	Burn Rate			
4	3,750.00	Thrust			
5					
6					
7					
8	Time	Mass	Accel	Velocity	Distance
9					
10	0.00	360.00	0	0.00	0.00
11	0.10	359.75	10.42	1.04	0.05
12	0.20	359.50	10.42	2.08	0.21
13	0.30	359.25	10.43	3.13	0.47
14	0.40	359.00	10.44	4.17	0.83
15	0.50	358.75	10.45	5.22	1.30
16	0.60	358.50	10.45	6.26	1.88
17	0.70	358.25	10.46	7.31	2.56
18	0.80	358.00	10.47	8.35	3.34

1326	131.60	31.00	120.00	3,672.66	151,675.44
1327	131.70	30.75	120.97	3,684.75	152,043.31
1328	131.80	30.50	121.95	3,696.95	152,412.39
1329	131.90	30.25	122.95	3,709.24	152,782.70
1330	132.00	30.00	123.97	3,721.64	153,154.24
1331					

$$Mass = (Mass_{prior\ row}) - (Burn\ Rate)(\Delta t)$$

$$Accel = (Thrust) / (Mass_{prior\ row})$$

$$Velocity = (Velocity_{prior\ row}) + (Accel_{current\ row})(\Delta t)$$

$$Dist = (Dist_{prior\ row}) + (Velocity_{prior\ row})(\Delta t) + \frac{1}{2}(Accel_{current\ row})(\Delta t)^2$$