Chapter 10 - Rotation of a Rigid Object About a Fixed Axis

Note Title 5/2/2005

1. (a). $\omega_f = \omega_i + \alpha t$, $\omega_i = 0$

$\omega_f = 12.0 = \alpha(3.0)$, $\alpha = 4.00 \text{ rad/sec}^2$

(b). $\omega_f^2 = 2\alpha\Delta\theta$, $\Delta\theta = \dfrac{(12.0)^2}{2(4.0)} = 18.0 \text{ rad}$

2. (a) $\omega_{earth} = \dfrac{2\pi \text{ rad}}{(365 d)\left(\frac{24 hr}{d}\right)\left(\frac{60 min}{hr}\right)\left(\frac{60 sec}{min}\right)}$

$= 1.99 \times 10^{-7} \text{ rad/sec.}$

(b). $F = \dfrac{G m_e m_m}{R^2}$, $\dfrac{F}{m_m} = a_{moon} = \dfrac{G m_e}{R^2} = \dfrac{v^2}{R} = R\omega^2$

$\therefore \omega_{moon} = \sqrt{\dfrac{G m_e}{R^3}}$

$= \sqrt{\dfrac{(6.67 \times 10^{-11} N \cdot m^2/kg^2)(5.98 \times 10^{24} kg)}{(3.84 \times 10^8 m)^3}}$

$= \sqrt{0.704 \times 10^{-11} N/kg \cdot m} = \sqrt{7.04 \times 10^{-12} \dfrac{kg \cdot m/sec^2}{kg \cdot m}}$

$= 2.65 \times 10^{-6} \text{ rad/sec}$

7. $\theta(t) = 5.00 + 10.0t + 2.00t^2$ rad.

$$\frac{d\theta}{dt} = 10.0 + 4.00t$$

$$\frac{d^2\theta}{dt} = 4.00$$

(a). $\therefore \theta(0) = 5.00$ rad
$\omega(0) = 10.0$ rad/sec
$\alpha(0) = 4.00$ rad/sec^2

(b). $\theta(3.00) = 53.0$ rad
$\theta'(3.00) = 22.0$ rad/sec
$\theta''(3.00) = 4.0$ rad/sec^2

15. (a). $w_f = w_i + \alpha t = 0 + 4(2) = 8.00$ rad/sec.

(b) 2m diameter $\Rightarrow$ 1.0 m radius

$V_T = rw = (1)(8) = 8.0$ m/sec.

$a_T = r\alpha = (1)(4) = 4.0$ m/sec^2

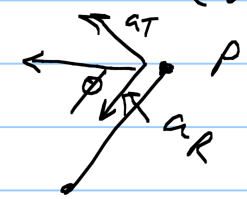
$a_r = \frac{v^2}{r} = rw^2 = 64.0$ m/sec^2

$\therefore a = \sqrt{a_T^2 + a_R^2} = \sqrt{64^2 + 4^2} = 64.1 \ m/sec^2$

for direction with respect to radius to point P :

$$\phi = \tan^{-1}\left(\frac{a_T}{a_r}\right) = \arctan\left(\frac{4}{64}\right)$$

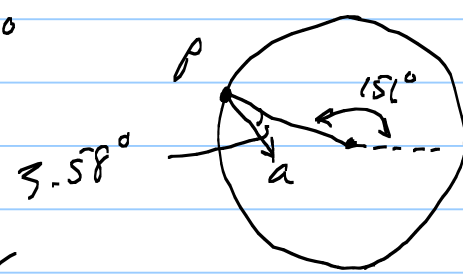
$$\phi = 3.58^\circ$$



(c) $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$$= 1 + 0 + \tfrac{1}{2}(4)(2)^2 = 9 \text{ rad}$$

$$\frac{9}{\pi} 180^\circ = 156^\circ$$



$\therefore$ acceleration vector is: $156^\circ + 180^\circ - 3.58^\circ$

$$= 332^\circ$$

23. (a) $I = \sum m_i r_i^2 = 4(3)^2 + 2(2^2) + 3(4)^2$

$$= 36 + 8 + 48$$

$$= 92 \ kg \cdot m^2$$

$$KE_{rot} = \tfrac{1}{2} I \omega^2 = \tfrac{1}{2}(92)(2.0)^2 = 184 \text{ J}$$

(b) $\frac{1}{2}(4)(3\cdot 2)^2 + \frac{1}{2}(2)(2\cdot 2)^2 + \frac{1}{2}(3)(4\cdot 2)^2$

$= 72 + 16 + 96 = 184\ J$

29. Treat each tire component as a hollow cylinder.

$I_{hollow\ cylinder} = \frac{1}{2} m (R_1^2 + R_2^2)$

Volume of hollow cylinder $= \pi (R_2^2 - R_1^2) h$

$\therefore$ Tread mass : $\pi (0.33^2 - 0.305^2)(0.20)(1.1\times 10^3)$
$= 10.972\ Kg$

Sidewall mass : $\pi (0.305^2 - 0.165^2)(0.635)(1.10\times 10^3)$
$= 1.44\ Kg$

$\therefore I_{tread} = \frac{1}{2}(10.972)(0.33^2 + 0.305^2) = 1.1$

$I_{sidewall} = \frac{1}{2}(1.44)(0.305^2 + 0.165^2) = 0.0868$

$\therefore 1.1 + 2(0.0868) = 1.27\ Kg\cdot m^2$

32. Net torque on big pulley must be 0.

$\therefore mg\, 3r = Tr$, $T =$ small pulley rope tension.

For The truck, 2T must balance it, and its weight along The ramp is $1500g \cos 45^\circ =$
$\frac{\sqrt{2}}{2} 1500 g = 750\sqrt{2}\, g = 2T.$

$\therefore T = 375\sqrt{2}\, g$, and $T = 3mg$

$\therefore 3mg = 375\sqrt{2}\, g$, $m = 125\sqrt{2}$ kg

35. Each tire has normal force of $1500g/4 = 375g$
For static friction, max force is:
$357g\, \mu_s = (0.8)(375g)$.

$\therefore$ max torque is: $(0.8)(375g)(0.6) = 1764$ N·m

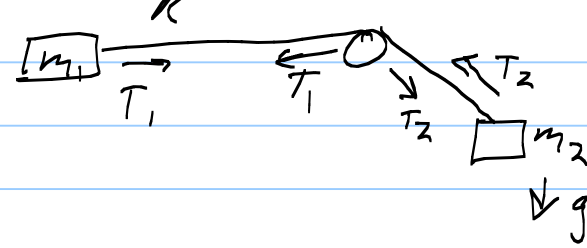
There are two driving wheels. $\therefore \frac{1764}{2} = 882$ N·m

39. (a) $T_1 - \mu_k m_1 g = m_1 a$ [1]

$m_2 g \sin\theta - T_2 - \mu_k m_2 g \cos\theta = m_2 a$ [2]

$(T_1 - T_2) R = I\alpha = -I\frac{a}{R} = \frac{1}{2} M R^2 \frac{a}{R} = -\frac{1}{2} M R a$

$\therefore T_1 - T_2 = -\frac{1}{2} M a$ [3]



Note: T_1 on pulley is (+), T_2 is (−), a to right,
so $R\alpha = -a$.

Adding [1] + [2],

$$T_1 - T_2 - \mu_k g(m_1 + m_2\cos\theta) + m_2 g\sin\theta = (m_1 + m_2)a$$

Using [3],

$$-\tfrac{1}{2}Ma - \mu_k g(m_1 + m_2\cos\theta) + m_2 g\sin\theta = (m_1 + m_2)a$$

$$\therefore \quad a = \frac{m_2 g\sin\theta - \mu_k g(m_1 + m_2\cos\theta)}{m_1 + m_2 + \frac{1}{2}M}$$

$$= \frac{(6.0)(9.8)(\frac{1}{2}) - (0.36)(9.8)(2.0 + 6.0(.866))}{2.0 + 6.0 + \frac{1}{2}(10)}$$

$$= 0.309 \ m/sec^2$$

(6) $T_1 - \mu_k m_1 g = m_1 a$, $T_1 = (0.36)(2.0)(9.8) + (2.0)(0.309)$

$\therefore T_1 = 7.67 \ N$

From [3], $T_1 - T_2 = -\frac{1}{2} Ma$

$$T_2 = 7.67 + \tfrac{1}{2}(10)(0.309)$$

$$= 9.22 \text{ N}$$

41. $I_{wheel} = MR^2 = (1.80 \text{ kg})(0.32 \text{ m})^2 = 0.184 \text{ kg}\cdot\text{m}^2$

(a) $\sum \tau = I\alpha = (0.184)(4.50 \text{ rad/sec}^2)$

Resistive torque $= (120 \text{ N})(0.32)$
Chain torque $= (x)(0.045)$

$$\therefore 0.045x - (120)(0.32) = (0.184)(4.50)$$

$$x = \frac{(0.184)(4.50) + (120)(0.32)}{0.045}$$

$$= 872 \text{ N} \quad \text{(for 9.00 cm sprocket)}$$

(b) For 5.60 cm sprocket,

$$x = (872 \text{ N})\left(\frac{0.045}{0.028}\right) = 1.40 \times 10^3 \text{ N}$$

43. From equations & formula Ex. 10.15,

$$V_f = \left[\frac{2(\Delta m)gh}{m_1 + m_2 + \frac{I}{R^2}}\right]^{\frac{1}{2}}$$

Here, $\Delta m = 5.00$ kg, $h = 1.50$ m, $m_1 = 15.0$ kg, $m_2 = 10.0$ kg, $I = \frac{1}{2}MR^2$, $M = 3.00$ kg

$$= \left[\frac{2(\Delta m)gh}{m_1 + m_2 + \frac{1}{2}M}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2(5.0)(9.8)(1.5)}{10 + 15 + 1.5}\right]^{\frac{1}{2}} = 2.36 \text{ m/sec}$$

49. (a). ΔU for center of mass $= MgR$

Energy entirely rotational at bottom, so

$MgR = \frac{1}{2}I\omega^2$. $I_{c.m.} = \frac{1}{2}MR^2$. About pivot, $I = I_{cm} + MD^2 = \frac{1}{2}MR^2 + MR^2$

$$= \frac{3}{2}MR^2$$

$\therefore MgR = \frac{1}{2}\left(\frac{3}{2}MR^2\right)\omega^2$, $\omega^2 = \frac{4}{3}\frac{g}{R}$, $\omega = 2\sqrt{\frac{g}{3R}}$

$\therefore V = \omega R = 2\sqrt{gR/3}$

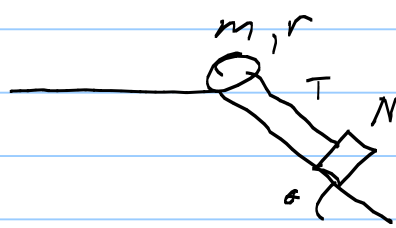
(b) $\omega(2R) = \left(2\sqrt{\frac{g}{3R}}\right)(2R) = 4\sqrt{gR/3}$

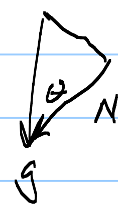
(c) $I_{loop} = mR^2$, so $I = mR^2 + mR^2 = 2mR^2$

$\therefore MgR = \frac{1}{2}(2mR^2)\omega^2$, $\omega = \sqrt{g/R}$

$v = R\omega = \sqrt{gR}$ for c.m.

63.





$\therefore$ using $v^2 = 2ad$, get a

$$Mg\sin\theta - T - \mu Mg\cos\theta = Ma$$

$$Tr = I\alpha = \left(\frac{1}{2}mr^2\right)\frac{a}{r}$$

$$\therefore T = \frac{1}{2}ma$$

$$\therefore Mg(\sin\theta - \mu\cos\theta) - \frac{1}{2}ma = Ma$$

$$\therefore \frac{2Mg(\sin\theta - \mu\cos\theta)}{m + 2M} = a$$

$$\therefore v^2 = 2\left(\frac{2Mg(\sin\theta - \mu\cos\theta)}{m+2M}\right)d$$

$$= \frac{4Mgd(\sin\theta - \mu\cos\theta)}{m+2M}$$

$$v = 2\sqrt{\frac{Mgd(\sin\theta - \mu\cos\theta)}{m+2M}}$$

(b) From above,

$$a = \frac{2Mg(\sin\theta - \mu\cos\theta)}{m+2M}$$

68. Let T = tension of string

Torque from string : TR

Torque of friction : τ

$\therefore TR - \tau = I\alpha$ [1], $I = \frac{1}{2}M\left(R^2 + \left(\frac{R}{2}\right)^2\right) = \frac{5}{8}MR^2$

For mass m: $mg - T = ma$ [2]

Also, $y = \frac{1}{2}at^2$, and $R\alpha = a$

$\therefore\ a = \frac{2y}{t^2}$, $\alpha = \frac{2y}{Rt^2}$

From [2], $T = mg - ma = mg - m\left(\frac{2y}{Rt^2}\right)$

From [1], $T = Rmg - Rm\frac{2y}{t^2} - I\alpha$

$$= Rmg - \frac{Rm\,2y}{t^2} - \frac{5}{8}MR^2\left(\frac{2y}{Rt^2}\right)$$

$$= Rmg - \frac{Rm\,2y}{t^2} - \frac{5}{4}MR\frac{y}{t^2}$$

$$= R\left[m\left(g - \frac{2y}{t^2}\right) - \frac{5My}{4t^2}\right]$$