Chapter 12 - Static Equilibrium and Elasticity

1. Since in equilibrium, force upward on bat by player must equal in magnitude the force of gravity. \[ F = 80 \text{ N upward} \]

   Total torque must also be 0. Torque of gravity is \( (10 \text{ N})(0.6 \text{ m}) = 6.0 \text{ N-m (clockwise)} \).

   \[ \therefore \text{Torque by player on bat} = 6.0 \text{ N-m counterclockwise} \]

9. Break up to track into small subsections of length \( \Delta x \).

   Assume uniform density. This will cancel in the equation:

   \[ m \Delta V_1 + m \Delta V_2 + \ldots + m \Delta V_n = m \Delta V_1 + m \Delta V_2 + \ldots + m \Delta V_n \]

   Where \( \Delta V_i \) = the volume of a subsection.

   \[ \Delta V_i = (\Delta x_i)(0.05) f(p_i), \text{where } p_i \text{ is a point in the interval of } \Delta x_i, \text{ and } f(x) = \frac{(x-3)^2}{9} \]

   \[ \lim_{\Delta x_i \to 0} \sum_{i=1}^{n} (\Delta x_i)(0.05) f(p_i) = \text{volume of } \]

   \[ \text{track} = \int_0^3 \left( \frac{(x-3)^2}{9} \right) \, dx = \left( \frac{0.05}{27} \right) \left( x^3-3x^2 \right) \bigg|_0^3 = \]

   \[ 0 - \left( \frac{0.05}{27} \right)(-27) = 0.05 \text{ m}^3 \]
For the numerator, \[ \int_0^3 \frac{(0.05) (x-3)^2}{9} \, dx = \]
\[ 0.05 \int_0^3 \frac{x^3 - 6x^2 + 9x}{9} \, dx = \]
\[ 0.05 \left( \frac{x^4}{4} - 2x^3 + \frac{9}{2} x^2 \right) \bigg|_0^3 = \]
\[ 0.05 \left( \frac{81}{4} - 54 + \frac{81}{2} \right) = \frac{0.05 \left(6.75\right)}{9} = 0.0375 \]

\[ \therefore \text{C.G.} = \frac{0.0375}{0.05} = 0.75 \text{ m} \]

15. (a) Let \( f_x = \text{friction force, } f_n = \text{normal force at point of contact.} \)

Let \( \mathbf{N} = \text{force of nail on hammer} \)

(By Newton's 3rd law \( \mathbf{a} = -\text{force hammer on nail} \))

\[ \therefore \mathbf{N}_x = 150 + f_x, \quad \mathbf{N}_y = f_n \]

From torques,
\[ (150 \text{ N})(30.0 \text{ cm}) = (f_y)(5.00 \text{ cm}) \]

\[ \therefore f_y = 900 \text{ N} \]
\[ N \cos 30^\circ = N_y \]

\[ N = \frac{900}{\cos 30^\circ} = 1.04 \times 10^3 \text{ N} \]

Directed 60° from horizon (counterclockwise)

(6) \[ N_x = N \sin 30^\circ = 520 \text{ N} \]
\[ \therefore f_x = 520 \times \sin 30^\circ = 320 \text{ N} \]

\[ \therefore \text{ from } f_x \text{ and } f_y \]
\[ \text{force} = (320 \text{ N})\hat{i} + (900 \text{ N})\hat{j} \]