Chapter 15 - Fluid Mechanics

Note Title 3/13/2006

2. **2.** Find the order of magnitude of the density of the nucleus of an atom. What does this result suggest concerning the structure of matter? Model a nucleus as protons and neutrons closely packed together. Each has mass 1.67×10^{-27} kg and radius on the order of 10^{-15} m.

$V = 4/3\,\pi r^3 \simeq 4r^3 = 4(10^{-15}\,m)^3 = 4 \times 10^{-45}\,m^3$

$\therefore \rho = \dfrac{1.67 \times 10^{-27}\,kg}{4 \times 10^{-45}\,m^3} = 4 \times 10^{17}\,kg/m^3$

$\therefore \rho_{nucleus} \simeq 10^{18}\,kg/m^3$

Since most solids have $\rho \simeq 10^3\,kg/m^3$,

$\therefore$ The density of space external to the nucleus up to the "edge" of the atom is very small in order to get an average density of the atom to be $\simeq 10^3\,kg/m^3$.

5. **5.** What is the total mass of the Earth's atmosphere? (The radius of the Earth is 6.37×10^6 m, and atmospheric pressure at the surface is 1.013×10^5 N/m^2.)

Surface area $= 4\pi r^2 = 4\pi(6.37 \times 10^6\,m)^2$

$1.013 \times 10^5\,N/m^2 = \dfrac{F_{total}}{4\pi(6.37 \times 10^6\,m)^2}$

$\therefore$ Weight = $F_{total} = 4\pi(6.37\times10^{6})^{2}(1.013\times10^{5})N$

$= 5.17\times10^{19}\ N$

This is total weight at earth surface, where $g = 9.8$.

$\therefore$ Mass = $F/g = \frac{5.17\times10^{19}}{9.8} = 5.27\times10^{18}\ kg$

10.

10. (a) A very powerful vacuum cleaner has a hose 2.86 cm in diameter. With no nozzle on the hose, what is the weight of the heaviest brick that the cleaner can lift? (Fig. P14.10a) (b) **What If?** A very powerful octopus uses one sucker of diameter 2.86 cm on each of the two shells of a clam in an attempt to pull the shells apart (Fig. P14.10b). Find the greatest force the octopus can exert in salt water 32.3 m deep. *Caution:* Experimental verification can be in-

(a) Assume vacuum cleaner removes atmospheric pressure on one side of brick.
$\therefore$ Air pressure on other side of brick pushes it up to hose opening.

Total force of air pressure is:

$$PA = (1.013\times10^{5}\ Pa)\left(\pi\left(\frac{0.0286}{2}\right)^{2}\right)$$

$$= \underline{65.1\ N}$$

(b) Is all the octopus can do is remove the

water pressure outside the clam. Any pressure inside the clam will force the shells to open. $\rho_{seawater} = 1.03 \times 10^3$ (Table 15.1)

Pressure at 32.3 m is: $P_0 + \rho g h$

$\therefore (1.013 \times 10^5 \text{ Pa}) + (1.03 \times 10^3 \text{ kg/m}^3)(9.8)(32.3 \text{ m})$

$= 4.27 \times 10^5 \text{ Pa}$

$\therefore$ Total force $= (4.27 \times 10^5 \text{ Pa})\left(\pi \left(\frac{0.028}{2}\right)^2\right)$

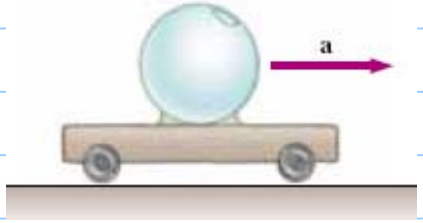
$= 274.5 \text{ N} = \underline{275 \text{ N}}$

11. Same analysis as in Example 15.4

$\therefore$ Total $F = \frac{1}{2}\rho g w l^2 = \frac{1}{2}(10^3 \text{ kg/m}^3)(9.8)(9.6)(2.4)^2$

$= \underline{2.71 \times 10^5 \text{ N}}$

13. A sealed spherical shell of diameter d is rigidly attached to a cart, which is moving horizontally with an acceleration a as in Figure P14.13. The sphere is nearly filled with a fluid having density ρ and also contains one small bubble of air at atmospheric pressure. Determine the pressure P at the center of the sphere.

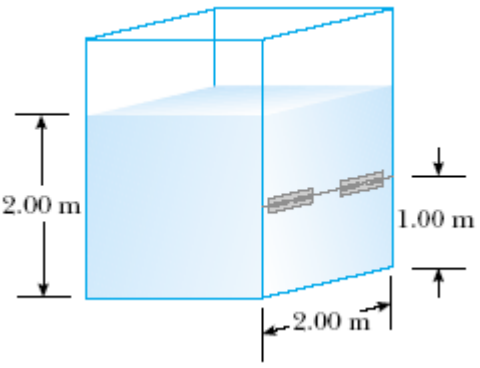


Pressure is a function of depth because of gravity, or an outside force. With the shell accelerating, the shell exerts a force on the liquid, which is transmitted uniformly throughout the liquid (Pascal's law).

It is easiest to analyze forces as if an observer were inside the shell. At the center, one sees a "gravitational" force of $\sqrt{g^2+a^2}$. $\therefore$ Pressure at center will be

$$P_0 + \left(\frac{d}{2}\right)\rho\sqrt{g^2+a^2}$$

14. The tank in Figure P14.14 is filled with water 2.00 m deep. At the bottom of one side wall is a rectangular hatch 1.00 m high and 2.00 m wide, which is hinged at the top of the hatch. (a) Determine the force the water exerts on the hatch. (b) Find the torque exerted by the water about the hinges.



Omit air pressure as it exists on both sides.

(a) $P = \rho g h$. Use axis so $h = 0$ at top and $h = 2.00$ m at bottom.

$$dF = P\,dA = \rho g h (2.00)\,dh$$

$$\therefore F = \int_{1.00}^{2.00} \rho g h (2.00)\, dh = \rho g h^2 \Big|_{1.00}^{2.00}$$

$$= (10^3)(9.8)[4.00 - 1.00]$$

$$= 2.94 \times 10^4 \text{ N, directed to right.}$$

(6) $d\tau = (dF)(\text{distance-to-hinge})$

$$= dF\,(h - 1.00)$$

$$= 2.00 \rho g h (h - 1.00)\, dh$$

$$\therefore \tau = \int_{1.00}^{2.00} 2.00 \rho g h (h - 1.00)\, dh$$

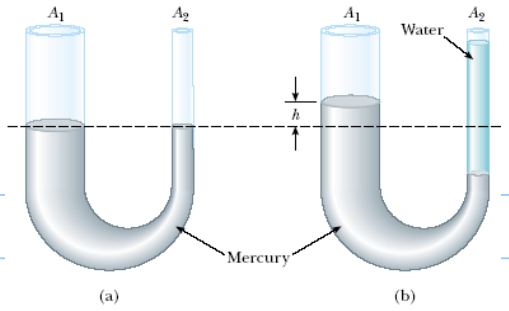
$$= 2.00 \rho g \int_1^2 h^2 - h\, dh$$

$$= 2.00 (10^3)(9.8) \left[\frac{h^3}{3} - \frac{h^2}{2}\right]\Big|_1^2$$

$$= (19.6 \times 10^3)\left(\frac{8}{3} - 2 - \frac{1}{3} + \frac{1}{2}\right)$$

$$= 1.63 \times 10^2 \text{ N-m}$$

18. Mercury is poured into a U-tube as in Figure P14.18a. The left arm of the tube has cross-sectional area A_1 of 10.0 cm^2, and the right arm has a cross-sectional area A_2 of 5.00 cm^2. One hundred grams of water are then poured into the right arm as in Figure P14.18b. (a) Determine the length of the water column in the right arm of the U-tube. (b) Given that the density of mercury is 13.6 g/cm^3, what distance h does the mercury rise in the left arm?



(a) $\rho_{water} = 1\ g/cc$, so $100\ g \Rightarrow 100\ cc$

$\therefore (5.00\ cm^2) h_w = 100\ cc$, $h_w = \underline{20\ cm}$

(b) Pressure at any level is the same, otherwise not in equilibrium (i.e., $P_{left} = P_{right}$)

Let h_w = distance of Hg-H_2O interface below original level.

At Hg-H_2O interface, $P_{right} = P_0 + \frac{(100\ g)(9.8\ m/sec^2)}{(5.00\ cm^2)}$

At same level on other side,

$$P_{left} = P_0 + \rho_{Hg} h g + \rho_{Hg} g h_w$$

Volume of displaced Hg is same, so

$(h_w)(5.00) = h(10.0)$, $\therefore h_w = 2h$

$$\therefore P_{left} = P_0 + \rho_{Hg} h g + \rho_{Hg} g (2h)$$

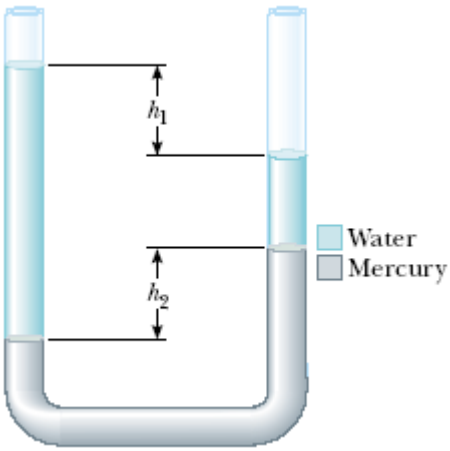
$$\therefore P_{left} = P_0 + 3h\rho_{Hg}g = P_0 + \left(\frac{100\,g}{5\,cm^2}\right)g$$

$$\therefore 3h\rho_{Hg} = \frac{100\,g}{5\,cm^2}$$

$$h = \left(\frac{100\,g}{5\,cm^2}\right)\cdot\frac{1}{3(13.6\,g/cm^3)}$$

$$= \underline{0.490\,cm}$$

19. A U-tube of uniform cross-sectional area, open to the atmosphere, is partially filled with mercury. Water is then poured into both arms. If the equilibrium configuration of the tube is as shown in Figure P14.20, with $h_2 = 1.00$ cm, determine the value of h_1.



Pressure at h_2 (H_2O-Hg interface same for both left and right sides. Ignore atmospheric pressure as it is present on both sides. Let x = distance of H_2O between h_1 and h_2.

$$\therefore \rho_{H_2O}gh_1 + \rho_{H_2O}gx + \rho_{H_2O}gh_2 = \rho_{H_2O}gx + \rho_{Hg}gh_2$$

$$\therefore \rho_{H_2O}(h_1 + h_2) = \rho_{Hg} h_2$$

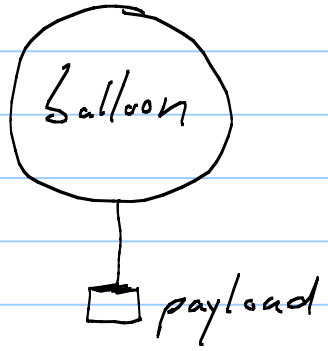
$$\therefore h_1 = \frac{\rho_{Hg}}{\rho_{H_2O}} h_2 - h_2 = h_2\left(\frac{\rho_{Hg}}{\rho_{H_2O}} - 1\right)$$

$$h_2 = 1.00\,cm, \therefore h_1 = \left(\frac{\rho_{Hg}}{\rho_{H_2O}} - 1\right) cm$$

$$\rho_{Hg} = 13.6\,g/cc, \rho_{H_2O} = 1.00\,g/cc$$

$$\therefore h_2 = \underline{\underline{12.6\,cm}}$$

20. (a) A light balloon is filled with 400 m^3 of helium. At 0°C, the balloon can lift a payload of what mass? (b) **What If?** In Table 14.1, observe that the density of hydrogen is nearly half the density of helium. What load can the balloon lift if filled with hydrogen?



(a) $B - F_{balloon} - F_{payload} = 0$

Since at max payload, no acceleration of system.

$$B = \rho_{air} V_g = (1.29\,Kg/m^3)(400\,m^3)g$$

$F_{balloon} = \rho_{He} V g = (1.79 \times 10^{-1} \text{ Kg/m}^3)(400 \text{ m}^3) g$

$F_{payload} = mg$

$\therefore (1.29 - 0.179)(400) = m$

$\therefore m = \underline{444 \text{ Kg}}$

(6) B is same. $F_{balloon} = \rho_{H} V g$

$= (0.0899)(400) g$

$\therefore (1.29 - 0.0899)(400) = m = \underline{480 \text{ Kg}}$

25. A cube of wood having an edge dimension of 20.0 cm and a density of 650 kg/m^3 floats on water. (a) What is the distance from the horizontal top surface of the cube to the water level? (b) How much lead weight has to be placed on top of the cube so that its top is just level with the water?

(a) Volume of cube: $(2.00 \times 10^{-1})^3 = 8.00 \times 10^{-3} \text{ m}^3$

Mass of cube: $(650 \text{ Kg/m}^3)(8.00 \times 10^{-3} \text{ m}^3) = 5.20 \text{ kg}$

$\therefore$ Mass H_2O displaced $= 5.20$ kg

$\rho_{H_2O} = 1 \text{ g/cc} = \frac{1 \text{ g}}{(10^{-2} \text{ m})^3} = 1 \text{ g}/10^{-6} \text{ m}^3 = 1 \text{ kg}/10^{-3} \text{ m}^3$

$\therefore$ Vol. of H_2O displaced: $\frac{5.20\,kg}{1\,kg/10^{-3}\,m^3} = 5.2\times10^{-3}\,m^3$

Area of cube: $(2.0\times10^{-1})^2 = 4\times10^{-2}\,m^2$

Dimension of H_2O displaced: $Ah = (4\times10^{-2}\,m^2)h$

$$\therefore h = \frac{5.2\times10^{-3}\,m^3}{4.0\times10^{-2}\,m^2} = 1.3\times10^{-1}\,m = 13\,cm$$

$\therefore 20 - 13 = \underline{7\,cm}$ = height of exposed cube

(b) You want weight in a $(20\,cm)^3$ volume to equal weight of H_2O.

From (a), $\rho_{H_2O} = 10^3\,kg/m^3$, Mass cube = 5.2 kg

$\therefore (5.2 + x)\,g = (8.00\times10^{-3}\,m^3)(10^3\,kg/m^3)\,g$

$\therefore 5.2\,kg + x = 8.00\,kg$, $x = \underline{2.80\,kg\ lead}$

26. To an order of magnitude, how many helium-filled toy balloons would be required to lift you? Because helium is an irreplaceable resource, develop a theoretical answer rather than an experimental answer. In your solution state what physical quantities you take as data and the values you measure or estimate for them.

Assume your weight is 70 Kg.
Helium balloons therefore need to displace 70 Kg of air.
Air at STP has $\rho \simeq 1.3\ kg/m^3$.
$\therefore$ Need about $50\ m^3$ of air.
Assume diameter of balloon to be $\simeq 30$ cm.
$\therefore$ Vol. of one balloon is $\frac{4}{3}\pi (0.15)^3$
$\simeq 0.01\ m^3$
$\therefore \frac{50}{0.01} = 5{,}000$ balloons $\simeq 10^4$

Balloon material & helium have some weight, but assume negligible, and in any case, will be compensated by an additional 5,000 balloons.
$\therefore \simeq 10^4$ balloons.

27. A plastic sphere floats in water with 50.0 percent of its volume submerged. This same sphere floats in glycerin with 40.0 percent of its volume submerged. Determine the densities of the glycerin and the sphere.

In water $\rho_{sphere} V g = \rho_{H_2O} (\frac{1}{2} V) g$, $\therefore \rho_{sphere} = \frac{1}{2} \rho_{H_2O}$

In glycerin $\rho_{sphere} V g = \rho_{glycerin} (0.40\ V) g$,

$\therefore \rho_{sphere} = \frac{1}{2}\rho_{H_2O} = \frac{2}{5}\rho_{glycerin}$, $\rho_{glycerin} = \frac{5}{4}\rho_{H_2O}$

28. A frog in a hemispherical pod (Fig. P14.34) just floats without sinking into a sea of blue-green ooze with density 1.35 g/cm^3. If the pod has radius 6.00 cm and negligible mass, what is the mass of the frog?

Weight of frog = weight of ooze displaced by pod.

Pod volume = $\frac{1}{2}\left(\frac{4}{3}\pi (6.00)^3\right) = 452\ cm^3$

Mass of ooze: $(452\ cm^3)(1.35\ g/cm^3) = 611\ g$

$\therefore$ Mass of frog = $\underline{611\ g}$

29. How many cubic meters of helium are required to lift a balloon with a 400-kg payload to a height of 8 000 m? (Take $\rho_{He} = 0.180\ kg/m^3$.) Assume that the balloon maintains a constant volume and that the density of air decreases with the altitude z according to the expression $\rho_{air} = \rho_0 e^{-z/8\,000}$, where z is in meters and $\rho_0 = 1.25\ kg/m^3$ is the density of air at sea level.

Let V = vol. of helium needed.

At 8,000 m, weight of balloon + payload = weight of displaced air.

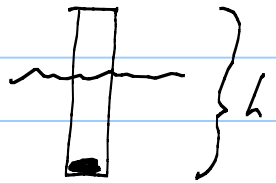
Air density: $\rho_0 e^{-\frac{8000}{8000}} = (1.25)(0.368) = 0.460\ kg/m^3$

$\therefore \rho_{He} V + 400\ kg = V(0.460)$, $V = \frac{400}{0.460 - 0.180}$

$= \underline{1.43 \times 10^3\ m^3}$

30. A long cylindrical tube of radius r is weighted on one end so that it floats upright in a fluid having a density ρ. It is pushed downward a distance x from its equilibrium position and then released. Show that the tube will execute simple harmonic motion if the resistive effects of the water are neglected, and determine the period of the oscillations.

Poorly worded question.



Assume x is ≤ height of cylinder above the fluid, because if top of cylinder is at surface of liquid, simple harmonic motion will not occur. The bouyant force is constant at any depth (assuming incompressible liquid), so you will just get simple acceleration.

∴ At a displacement of x, volume of liquid displaced is $x\pi r^2$, so mass of liquid displaced is $\rho\pi r^2 x$, and ∴ weight is $\rho g\pi r^2 x$.

By Archimedes' principle, the bouyant force is up and is equal to weight of displaced fluid.

Assume x is down, ∴ $F_B = -\rho g\pi r^2 x$

∴ This is a restoring force and a function of displacement, and so motion is

simple harmonic motion.

Mass of cylinder is m_c

$\therefore F_B = -\rho g \pi r^2 x = m_c a$

For simple harmonic motion,

$$T = 2\pi \sqrt{\frac{m}{K}}$$

Here, $K = \rho g \pi r^2$, $T = 2\pi \sqrt{\frac{m_c}{\rho g \pi r^2}}$

---

31. A bathysphere used for deep-sea exploration has a radius of 1.50 m and a mass of 1.20×10^4 kg. To dive, this submarine takes on mass in the form of seawater. Determine the amount of mass the submarine must take on if it is to descend at a constant speed of 1.20 m/s, when the resistive force on it is 1 100 N in the upward direction. The density of seawater is 1.03×10^3 kg/m^3.

Constant speed implies zero force, so assume something gave object a downward motion. When take on water, density is s.t. forces are in equilibrium. Let m = mass of H_2O.

$\therefore (1.2 \times 10^4 + m)g = F_B + 1100$, F_B = bouyant force

$$F_B = g\frac{4}{3}\pi(1.50)^3\rho_{\text{sea water}} = \frac{4}{3}\pi(1.50)^3(1.03\times10^3)g$$

$$= 143\times10^3$$

$$\therefore (1.2\times10^4 + m)g = 144\times10^3$$

$$m = \frac{1.44\times10^5}{9.8} - 1.2\times10^4 = \underline{\underline{2.67\times10^3\ \text{kg}}}$$

32. The United States possesses the eight largest warships in the world—aircraft carriers of the *Nimitz* class—and is building two more. Suppose one of the ships bobs up to float 11.0 cm higher in the water when 50 fighters take off from it in 25 min, at a location where the free-fall acceleration is 9.78 m/s^2. Bristling with bombs and missiles, the planes have average mass 29 000 kg. Find the horizontal area enclosed by the waterline of the \$4-billion ship. By comparison, its flight deck has area 18 000 m^2. Below decks are passageways hundreds of meters long, so narrow that two large men cannot pass each other.

Weight of planes: $(50)(2.9\times10^4)g = 1.42\times10^7\ \text{N}$

$\rho_{\text{sea water}} = 1.03\times10^3\ \text{kg/m}^3$

$$(\text{Area})(0.11\text{ m})(1.03\times10^3\ \text{kg/m}^3)g = (50)(2.9\times10^4)g$$

$$\text{Area} = \frac{(50)(2.9\times10^4)}{(0.11)(1.03\times10^3)} = \underline{\underline{1.28\times10^4\ \text{m}^2}}$$

33. (a) $20.0\ L/min = 20 \times 10^3\ cc/min$

Hose area: $\pi (1.0\ cm)^2 = \pi\ cm^2$

$\therefore \pi V = 20 \times 10^3$, $V = 6.37 \times 10^3\ cm/min$

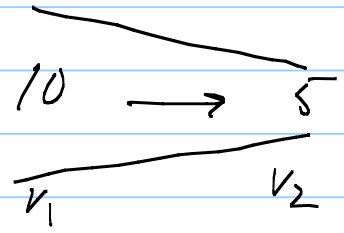
$= 1.06 \times 10^2\ cm/sec.$

$= 1.06\ m/sec$

(b) $A_1 V_1 = A_2 V_2$. $\therefore (2.0)^2 (1.06) = (1.0)^2 V_2$

$V_2 = 4.24\ m/sec.$

34. A horizontal pipe 10.0 cm in diameter has a smooth reduction to a pipe 5.00 cm in diameter. If the pressure of the water in the larger pipe is 8.00×10^4 Pa and the pressure in the smaller pipe is 6.00×10^4 Pa, at what rate does water flow through the pipes?



$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$, $A_1 V_1 = A_2 V_2$

$\therefore P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho \left(\frac{A_1}{A_2} V_1\right)^2 = P_2 + \frac{1}{2}\rho \frac{A_1^2}{A_2^2} V_1^2$

$\therefore \frac{1}{2}\rho V_1^2 \left(1 - \frac{A_1^2}{A_2^2}\right) = P_2 - P_1$, $\frac{A_1}{A_2} = \frac{100}{25} = 4$

$\therefore \frac{1}{2}\rho V_1^2 (1 - 16) = P_2 - P_1$,

$V_1^2 = \frac{2(P_1 - P_2)}{15\rho}$, $V_1 = \sqrt{\frac{2(P_1 - P_2)}{15\rho}}$

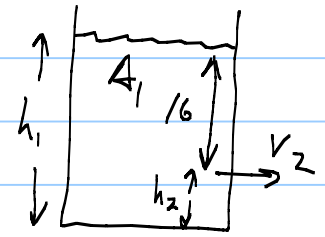
$$\therefore v_1 = \sqrt{\frac{2(2.00\times10^4)}{15\cdot10^3}} = \sqrt{\frac{40}{15}} = 1.63 \text{ m/sec}$$

$$\therefore v_2 = v_1\left(\frac{A_1}{A_2}\right) = (1.63)\left(\frac{100}{25}\right) = 6.53 \text{ m/sec}$$

In terms of mass flowing, $\rho A_1 v_1 =$

$$(10^3)\left(\frac{\pi(0.1)^2}{4}\right)(1.63) = \underline{\underline{12.8 \text{ kg/sec}}}$$

35.]A large storage tank, open at the top and filled with water, develops a small hole in its side at a point 16.0 m below the water level. If the rate of flow from the leak is equal to 2.50×10^{-3} m^3/min, determine (a) the speed at which the water leaves the hole and (b) the diameter of the hole.



(a). Assume pin hole area << top area storage tank.

$A_1 v_1 = A_2 v_2$, so $A_2 << A_1$, so $v_1 = \frac{A_2}{A_1} v_2$,

so $\frac{A_2}{A_1} << 1$, so $v_1 \approx 0$.

Also, $P = P_0$ for top of container and outside around v_2.

Using $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

$$P_0 + \rho g(h_1 - h_2) = P_0 + \frac{1}{2}\rho v_2^2$$

$$\therefore (10^3)(9.8)(16) = \tfrac{1}{2}(10^3)(v_2^2)$$

$$v_2 = \sqrt{2(9.8)(16)} = 17.7 \text{ m/sec.}$$

(b) Flow rate $= Av = A(17.7 \text{ m/sec})$

$$= 2.50 \times 10^{-3} \text{ m}^3/\text{min} = \frac{2.50 \times 10^{-3} \text{ m}^3/\text{min}}{60 \text{ sec/min}}$$

$$= 4.17 \times 10^{-5} \text{ m}^3/\text{sec}$$

$$\therefore A = \frac{4.17 \times 10^{-5} \text{ m}^3/\text{sec}}{17.7 \text{ m/sec}} = 2.35 \times 10^{-6} \text{ m}^2$$

$$\frac{\pi d^2}{4} = 2.35 \times 10^{-6} \text{ m}^2,$$

$$d = \sqrt{\frac{4(2.35 \times 10^{-6})}{\pi}} = 1.73 \times 10^{-3} \text{ m}$$

$$= \underline{1.73 \text{ mm}}$$

36. Through a pipe 15.0 cm in diameter, water is pumped from the Colorado River up to Grand Canyon Village, located on the rim of the canyon. The river is at an elevation of 564 m, and the village is at an elevation of 2 096 m.

(a) What is the minimum pressure at which the water must be pumped if it is to arrive at the village? (b) If 4 500 m^3 are pumped per day, what is the speed of the water in the pipe? (c) What additional pressure is necessary to deliver this flow? *Note*: Assume that the free-fall acceleration and the density of air are constant over this range of elevations.

(a) Assume pipe is open at other end, and velocities are zero.

$$\therefore P = \rho g \Delta h = (10^3)(9.8)(2096 - 564)$$

$$= 1.50 \times 10^7 \text{ Pascal}$$

(b) $\Delta V = 4.50 \times 10^3 \text{ m}^3/\text{day}$

$$A = \pi (7.5 \times 10^{-2} \text{ m})^2$$

$$\therefore V = \frac{4.5 \times 10^3 \text{ m}^3/\text{day}}{\pi (7.5 \times 10^{-2})^2 (24 \text{ hr/day})(60 \text{ min/hr})(60 \text{ sec/min})}$$

$$= 2.95 \text{ m/sec}$$

(c) From $P + \frac{1}{2}\rho v^2 + \rho g h$, assume v at bottom is 0, top & bottom at P_0, have already calculated $\rho g \Delta h$, so additional

velocity contributes $\frac{1}{2}\rho V^2$ to pressure requirement.

$\therefore \frac{1}{2}(10^3 \text{ Kg/m}^3)(2.95 \text{ m/sec})^2 = 4.34 \times 10^3 \text{ Pascal}$

38. Old Faithful Geyser in Yellowstone Park (Fig. P14.46) erupts at approximately 1-h intervals, and the height of the water column reaches 40.0 m. (a) Model the rising stream as a series of separate drops. Analyze the free-fall motion of one of the drops to determine the speed at which the water leaves the ground. (b) **What If?** Model the rising stream as an ideal fluid in streamline flow. Use Bernoulli's equation to determine the speed of the water as it leaves ground level. (c) What is the pressure (above atmospheric) in the heated underground chamber if its depth is 175 m? You may assume that the chamber is large compared with the geyser's vent.

(a) $V_f^2 = V_i^2 - 2gd$, $0 = V_i^2 - 2(9.8)(40.0)$,

$V_i = 28.0$ m/sec

(b) $P_i + \frac{1}{2}\rho V_i^2 + \rho g h_i = P_f + \frac{1}{2}\rho V_f^2 + \rho g h_f$.

Here, $P_i = P_f = P_0$, $V_f = 0$, so,

$\frac{1}{2}\rho V_i^2 = \rho g h_f$, $V_i^2 = 2gh_f$,

$V_i = \sqrt{2gh_f} = \sqrt{2(9.8)(40)} = 28.0$ m/sec.

(c) Assuming large chamber underground $\Rightarrow$

negligible velocity at bottom.

$\therefore P = P_0 + \frac{1}{2}\rho v^2 + \rho g(175)$ [h = 0 at bottom]

$\therefore P - P_0 = \frac{1}{2}(10^3)(28.0)^2 + \rho g(175)$

$\therefore P - P_0 = 2.11 \times 10^6$ Pascals

39. An airplane has a mass of 1.60×10^4 kg, and each wing has an area of 40.0 m^2. During level flight, the pressure on the lower wing surface is 7.00×10^4 Pa. Determine the pressure on the upper wing surface.

Weight of plane $= (9.8)(1.6 \times 10^4)$

$\therefore 2(7.00 \times 10^4 - P)(40.0) = 9.8(1.6 \times 10^4)$

$P = 6.80 \times 10^4$ Pascals

40. A Venturi tube may be used as a fluid flow meter (see Fig. 14.20). If the difference in pressure is $P_1 - P_2 = 21.0$ kPa, find the fluid flow rate in cubic meters per second, given that the radius of the outlet tube is 1.00 cm, the radius of the inlet tube is 2.00 cm, and the fluid is gasoline ($\rho = 700$ kg/m^3).

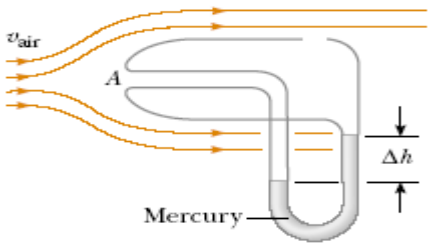
$A_1 v_1 = A_2 v_2$, $\therefore 4v_1 = v_2$

$\therefore P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho(16 v_1^2)$

$$\therefore v_1 = \sqrt{\frac{2(P_1 - P_2)}{15\rho}} = \sqrt{\frac{2(21.0\times10^3)}{15(700)}} = 2.0 \text{ m/sec}$$

$$\therefore A_1 v_1 = \pi r^2 (2.0) = \pi (.02)^2 (2.0) = 2.51\times10^{-3} \text{ m}^3/\text{sec}$$

41. A Pitot tube can be used to determine the velocity of air flow by measuring the difference between the total pressure and the static pressure (Fig. P14.49). If the fluid in the tube is mercury, density $\rho_{Hg} = 13\,600$ kg/m^3, and $\Delta h = 5.00$ cm, find the speed of air flow. (Assume that the air is stagnant at point A, and take $\rho_{air} = 1.25$ kg/m^3.)



$$\rho_{Hg} g \Delta h = P_A - P_0 \quad \text{(device open to air at top)}$$

Air stagnant at A $\Rightarrow v_A = 0$. $\therefore P_0 + \frac{1}{2}\rho_{air} v^2 = P_A$

$$\therefore P_A - P_0 = \tfrac{1}{2}\rho v^2. \quad \therefore \tfrac{1}{2}\rho_{air} v^2 = \rho_{Hg} g \Delta h,$$

$$\therefore v^2 = \frac{2\rho_{Hg} g \Delta h}{\rho_{air}}$$

$$v = \sqrt{\frac{2(13.6\times10^3)(9.8)(0.05)}{(1.25)}} = 103 \text{ m/sec}$$

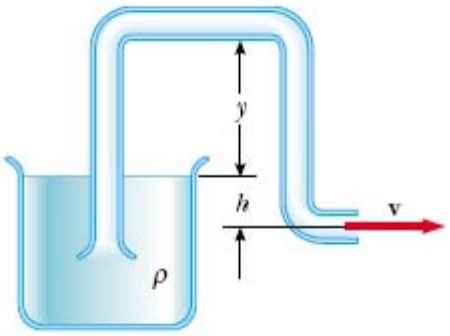
42. An airplane is cruising at an altitude of 10 km. The pressure outside the craft is 0.287 atm; within the passenger compartment the pressure is 1.00 atm and the temperature is 20°C. A small leak occurs in one of the window seals in the passenger compartment. Model the air as an ideal fluid to find the speed of the stream of air flowing through the leak.

Assume negligible velocity of air inside plane.

$\therefore P_0 + \frac{1}{2}\rho v^2 = P_i$, $v = \sqrt{\frac{(1.0 - 0.287)(1.013 \times 10^5)2}{1.29}}$

$= 335$ m/sec

43. A siphon is used to drain water from a tank, as illustrated in Figure P14.51. The siphon has a uniform diameter. Assume steady flow without friction. (a) If the distance $h = 1.00$ m, find the speed of outflow at the end of the siphon. (b) **What If?** What is the limitation on the height of the top of the siphon above the water surface? (For the flow of the liquid to be continuous, the pressure must not drop below the vapor pressure of the liquid.)



(a) At surface of water in tank, $P = P_0$, and assume speed of water level change negligible. At outlet, $P = P_0$.

$\therefore P_0 = P_0 + \frac{1}{2}\rho v^2 + \rho g(-h)$, $\rho g h = \frac{1}{2}\rho v^2$

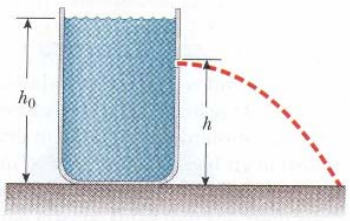
$\therefore v^2 = 2gh$, $v = \sqrt{2(9.8)(1.0)} = 4.43$ m/sec

(6) At atmosph. press., water can only climb, in a vacuum tube, $P_0 = \rho g y$,

$$\therefore y = \frac{1.013 \times 10^5}{(10^3)(9.8)} = \underline{\underline{10.3\ m}}$$

(760 mm for Hg)

45. A large storage tank is filled to a height h_0. The tank is punctured at a height h above the bottom of the tank (Fig. P15.45). Find an expression for how far from the tank the exiting stream lands.



Assume velocity of water level is negligible. Both tank and stream exposed to P_0!

$$\therefore P_0 + 0 + 0 = P_0 + \tfrac{1}{2}\rho v^2 - \rho g(h_0 - h)$$

$$\therefore \rho g(h_0 - h) = \tfrac{1}{2}\rho v^2,\ v^2 = 2g(h_0 - h),\ v = \sqrt{2g(h_0 - h)}$$

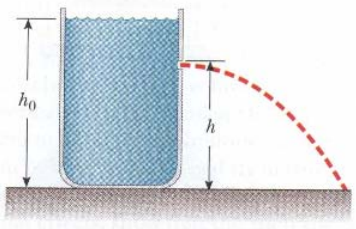
For projectile motion, time in flight for vertical drop from height $h = \frac{1}{2} g t^2$.

$$\therefore t = \sqrt{\frac{2h}{g}}$$

$\therefore$ Horiz dist. travelled $= vt$

$$vt = \sqrt{2g(h_0-h)}\sqrt{\frac{2h}{g}} = \underline{2\sqrt{h(h_0-h)}}$$

46. A hole is punched at a height h in the side of a container of height h_0. The container is full of water, as shown in Figure P15.45. If the water is to shoot as far as possible horizontally, (a) how far from the bottom of the container should the hole be punched? (b) Neglecting frictional losses, how far (initially) from the side of the container will the water land?



(a) From (a), $d(h) = 2\sqrt{h(h_0-h)}$

$d(h)$ is a max when $h(h_0-h) = h_0h - h^2$ is a max.

Let $f(h) = h_0h - h^2$

$\therefore f'(h) = h_0 - 2h$, $f''(h) = -2$

$\therefore f'(h) = 0 = h_0 - 2h$, $h = \frac{1}{2}h_0$

$\therefore$ at $\underline{h = \frac{1}{2}h_0}$, $f(h)$ is a max.

(b) at $h = \frac{1}{2}h_0$, $d(h) = 2\sqrt{\frac{h_0}{2}\left(\frac{h_0}{2}\right)} = \underline{h_0}$

51.]The true weight of an object can be measured in a vacuum, where buoyant forces are absent. An object of volume V is weighed in air on a balance with the use of weights of density ρ. If the density of air is ρ_{air} and the balance reads F_g', show that the true weight F_g is

$$F_g = F_g' + \left(V - \frac{F_g'}{\rho g}\right)\rho_{\text{air}}\, g$$

The bouyant force on the object is $\rho_{air} V g$, and must be added to F_g' since the scale, in air, subtracted it.

The weights used to measure F_g' were also influenced by air bouyancy. F_g' is the value of the weights, and on the scale, the two forces must balance. If B_v is bouyancy of object and B_w is bouyancy of weights, then

$$F_g' - B_w = F - B_v$$

$$\therefore F = F_g' - B_w + B_v$$

If the weights have volume V_w, then

$$F_g' = (\text{mass}_{wts})\, g, \quad \text{mass}_{wts} = \rho V_w,$$

$$\therefore F_g' = \rho V_w g, \quad V_w = \frac{F_g'}{\rho g}$$

$$\therefore B_w = \rho_{air} V_w g = \frac{F_g' \rho_{air}}{\rho g} \cdot g = \frac{F_g' \rho_{air}}{\rho}$$

$$\therefore F = F_g' - \frac{F_g' \rho_{air}}{\rho} + \rho_{air} V g$$

$$= F_g' + \left(V - \frac{F_g'}{\rho g} \right) \rho_{air} g$$

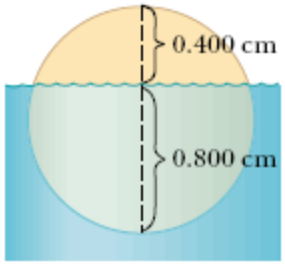
52. Evangelista Torricelli was the first person to realize that we live at the bottom of an ocean of air. He correctly surmised that the pressure of our atmosphere is attributable to the weight of the air. The density of air at 0°C at the Earth's surface is 1.29 kg/m^3. The density decreases with increasing altitude (as the atmosphere thins). On the other hand, if we assume that the density is constant at 1.29 kg/m^3 up to some altitude h, and zero above that altitude, then h would represent the depth of the ocean of air. Use this model to determine the value of h that gives a pressure of 1.00 atm at the surface of the Earth. Would the peak of Mount Everest rise above the surface of such an atmosphere?

Here, $P_0 = \rho_{air} g h = (1.29)(9.8)h = 1.013 \times 10^5$.

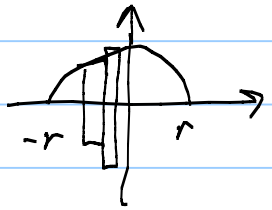
$\therefore h = 8.01 \times 10^3 \text{ m}$

Mt. Everest $> 29{,}000 \text{ ft} = 8.84 \times 10^3 \text{ m}$.

53. A wooden dowel has a diameter of 1.20 cm. It floats in water with 0.400 cm of its diameter above water (Fig. P14.58). Determine the density of the dowel.



Develop general formula for partial volume of sphere.



$y = \sqrt{r^2 - x^2}$, $y^2 = r^2 - x^2$

$\therefore$ area of each disk is $\pi(r^2 - x^2)$

Vol. of a disk is $\pi(r^2 - x^2)\Delta x$

Let d = distance from origin, $-r < d < r$.

$$\therefore \int_{-r}^{d} \pi(r^2 - x^2)\,dx = \pi\left(r^2 x - \frac{x^3}{3}\right)\Bigg|_{-r}^{d}$$

$$= \pi\left(r^2 d - \frac{d^3}{3} + r^3 - \frac{r^3}{3}\right) = \pi d\left(r^2 - \frac{d^2}{3}\right) + \frac{2}{3}\pi r^3$$

In this problem, $r = 0.600$ cm, $d = 0.200$ cm

$\therefore$ Weight of displaced water $= \rho_{H_2O}\, \text{Vol}_{\text{under}}\, g$

$$\text{Vol}_{\text{under}} = \pi(0.2)\left(0.6^2 - \frac{0.2^2}{3}\right) + \frac{2}{3}\pi(0.6)^3$$

$= 0.67\ cm^3$

$\therefore \rho_{H_2O} \, Vol_{under} = (10^{-3}\ kg/cc)(0.67\ cc)$

$= 6.7 \times 10^{-4}\ kg$ = mass displaced H_2O

$\therefore \rho_{dowel} = \dfrac{\text{mass displaced } H_2O}{\text{Vol of dowel}}$

$= \dfrac{6.7 \times 10^{-4}\ kg}{\frac{4}{3}\pi (6 \times 10^{-3}\ m)^3} = 726\ kg/m^3$

64. Show that the variation of atmospheric pressure with altitude is given by $P = P_0 e^{-\alpha y}$, where $\alpha = \rho_0 g / P_0$, P_0 is atmospheric pressure at some reference level $y = 0$, and ρ_0 is the atmospheric density at this level. Assume that the decrease in atmospheric pressure over an infinitesimal change in altitude (so that the density is approximately uniform) is given by $dP = -\rho g\, dy$, and that the density of air is proportional to the pressure.

$P = P_0 - \rho g h$, so $\dfrac{dP}{dh} = -\rho g$

Assuming $\rho \propto P$, then $\rho = kP$

$\rho = \rho_0$ when $P = P_0$, $\therefore k = \dfrac{\rho_0}{P_0}$

$\therefore \rho = \left(\dfrac{\rho_0}{P_0}\right) P$

$$\therefore \frac{dP}{dh} = -\left(\frac{\rho_0}{P_0}\right) P g \text{, or } \frac{dP}{P} = -\frac{\rho_0}{P} g\, dh$$

$$\therefore \int_{P_0}^{P} \frac{dP}{P} = -\frac{\rho_0}{P_0} g \int_0^h dh$$

$$\ln P \Big|_{P_0}^{P} = -\frac{\rho_0}{P_0} g h$$

$$\ln P - \ln P_0 = -\frac{\rho_0}{P_0} g h$$

$$\ln\left(\frac{P}{P_0}\right) = -\frac{\rho_0}{P_0} g h$$

$$\frac{P}{P_0} = e^{-\frac{\rho_0}{P_0} g h}$$

$$\therefore \text{Let } \alpha = \frac{\rho_0 g}{P_0}, \quad \therefore \underline{\underline{P = P_0 e^{-\alpha h}}}$$

C5. A cube of ice whose edges measure 20.0 mm is floating in a glass of ice-cold water with one of its faces parallel to the water's surface. (a) How far below the water surface is the bottom face of the block? (b) Ice-cold ethyl alcohol is gently poured onto the water surface to form a layer 5.00 mm thick above the water. The alcohol does not mix with the water. When the ice cube again attains hydrostatic equilibrium, what will be the distance from the top of the water to the bottom face of the block? (c) Additional cold ethyl alcohol is poured onto the water's surface until the top surface of the alcohol coincides with the top surface of the ice cube (in hydrostatic equilibrium). How thick is the required layer of ethyl alcohol?

(a) $\rho_{ice} V_{ice} g = \rho_{H_2O} V_{under} g$

$\therefore \dfrac{\rho_{ice}}{\rho_{H_2O}} = \dfrac{V_{under}}{V_{ice}}$

$\therefore \dfrac{0.917 \times 10^3}{1.00 \times 10^3} = \dfrac{V_{under}}{(20\ mm)^3}$

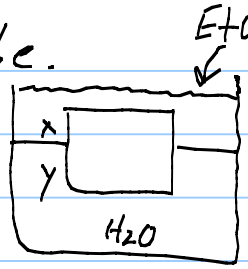
$V_{under} = 7336\ mm^3$, $Face = 400\ mm^2$,

$\therefore h(400) = 7336$, $h = 18.3\ mm$

$\therefore$ Bottom face 18.3 mm under surface

(b) $\rho_{EtOH} = 0.806 \times 10^3$, so ice should sink in pure ethanol.

∴ Assume ethanol covers ice cube.



$x + y = 20\ mm$

Pressure on cube on top: $\rho_{EtOH}\, g\,(5.0 - x)$

Pressure at level y at bottom of cube is:

$$\rho_{EtOH}\, g\,(5) + \rho_{H_2O}\, g\, y$$

For simplicity, let $s = 20\ mm$

Force of gravity on cube: $\rho_{ice}\, s^3\, g$

$$\therefore \rho_{ice}\, s^3 g + \left(\rho_{EtOH}\, g\,(5.0 - s + y)\right)(s^2) =$$

$$\left[\rho_{EtOH}\, g\,(\bar{5}_{mm}) + \rho_{H_2O}\, g\, y\right](s^2)$$

$$\rho_I s^3 + \rho_E (\bar{5} - s) s^2 + \rho_E\, y\, s^2 = \rho_E\, \bar{5}\,(s^2) + \rho_H\, y\, s^2$$

$$y\,(s^2\rho_H - s^2\rho_E) = s^2\left[s\rho_I - \bar{5}\rho_E + \bar{5}\rho_E - s\rho_E\right]$$

$$= s^3\left[\rho_I - \rho_E\right]$$

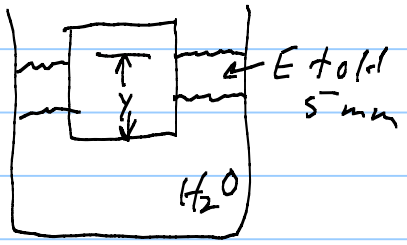
$$\therefore y = \frac{5(\rho_I - \rho_E)}{\rho_H - \rho_E} = 20\,mm\left(\frac{0.917 - 0.806}{1.00 - 0.806}\right)$$

$$= 11.4\,mm$$

But this means $x = 8.6$ mm, so that cube top face not submerged, as assumed.

$\therefore$ Assume cube top face above ethanol.

$\therefore$ Weight of cube $= \rho_I s^3 g$



Bouyant force is from pressure at bottom face of cube.

Pressure $= \rho_E g(5\,mm) + \rho_H g(y-5)$

$$\therefore \rho_I s^3 g = s^2\left[\rho_E g(5) + \rho_H g(y-5)\right] \qquad [1]$$

$$\rho_I s = 5\rho_E + y\rho_H - 5\rho_H$$

$$y = \rho_I s + 5(\rho_H - \rho_E) = (0.917)(20) + 5(1.0 - 0.806)$$

$$= 19.3\,mm$$

$\therefore y - 5 = \underline{14.3\ mm}$ = dist. from top of water to cube bottom face.

(c) In equation [1] in (6), substitute h for 5 mm and S (=20 mm) for y.

$$\therefore \rho_I S^3 g = S^2 [\rho_E g h + \rho_H g (S - h)]$$

$$\rho_I S = \rho_E h + S\rho_H - h\rho_H$$

$$h(\rho_H - \rho_E) = S(\rho_H - \rho_I)$$

$$h = S\left(\frac{\rho_H - \rho_I}{\rho_H - \rho_E}\right) = 20\left(\frac{1.0 - 0.917}{1.0 - 0.806}\right)$$

$= \underline{8.56\ mm}$ = Thickness of ethanol layer to cover cube.