Chapter 19 - Temperature

Note Title 11/12/2006

2. In a constant-volume gas thermometer, the pressure at 20.0°C is 0.980 atm. (a) What is the pressure at 45.0°C? (b) What is the temperature if the pressure is 0.500 atm?

(a) $P_1 V = nRT_1$ $\quad \therefore \quad \frac{P_2}{P_1} = \frac{T_2}{T_1}$

$P_2 V = nRT_2$

$$\therefore P_2 = 0.98 \left(\frac{273 + 45}{273 + 20} \right)$$

$$= \underline{\underline{1.06 \text{ atm}}}$$

(b) $\frac{P_2}{P_1} = \frac{T_2}{T_1}$, $\quad T_2 = T_1 \left(\frac{P_2}{P_1} \right)$

$$\therefore T_2 = (273 + 20) \left(\frac{0.500}{0.980} \right) = 149 \text{ K}$$

$$= \underline{-124^\circ C}$$

6. On a Strange temperature scale, the freezing point of water is − 15.0°S and the boiling point is + 60.0°S. Develop a *linear* conversion equation between this temperature scale and the Celsius scale.

$60 - (-15) = K\,100, \quad K = 0.75$

$\therefore {}^\circ S = (0.75)\,{}^\circ C + \alpha$

at $°C = 0°$, $°S = -15.0$

$\therefore \alpha = -15$

$\therefore °S = \frac{3}{4}°C - 15.0$

or $°S + 15 = \frac{3}{4}°C$, or $°C = \frac{4}{3}°S + 20$

9. A copper telephone wire has essentially no sag between poles 35.0 m apart on a winter day when the temperature is $-20.0°C$. How much longer is the wire on a summer day when $T_C = 35.0°C$?

$\frac{\Delta L}{L_i} = \alpha \Delta T$, $\alpha_{Cu} = 17 \times 10^{6}$

$\therefore \Delta L = L_i \alpha_{Cu} \Delta T = (35.0\,m)(17 \times 10^{-6}\,°C^{-1})(55°C)$

$= 32.7 \times 10^{-3}\,m = 32.7\,mm$

12. A brass ring of diameter 10.00 cm at 20.0°C is heated and slipped over an aluminum rod of diameter 10.01 cm at 20.0°C. Assuming the average coefficients of linear expansion are constant, (a) to what temperature must this combination be cooled to separate them? Is this attainable? (b) **What If?** What if the aluminum rod were 10.02 cm in diameter?

(a) $\alpha_{Brass} = 19 \times 10^{6}\ {}^{\circ}C^{-1}$, $\alpha_{Al} = 24 \times 10^{-6}\ {}^{\circ}C^{-1}$

Since Circumference $= \pi d$, $C_1 = \pi d_1$, $C_2 = \pi d_2$,

$\therefore \Delta C = \pi \Delta d.$

$\therefore \dfrac{\Delta C}{C} = \alpha \Delta T$, $\dfrac{\pi \Delta d}{\pi d} = \alpha \Delta T$, so

$\dfrac{\Delta d}{d} = \alpha \Delta T$, and only need to deal with diameter.

$$\therefore \frac{\Delta d_{Brass}}{d_{Brass}} = \alpha_B \Delta T, \quad \frac{\Delta d_{Al}}{d_{Al}} = \alpha_{Al} \Delta T \quad [1]$$

We need $d_{Brass} + \Delta d_{Brass} = d_{Al} + \Delta d_{Al}$,

or $10.0 + \Delta d_B = 10.01 + \Delta d_{Al}$,

or $\Delta d_B = 0.01 + \Delta d_{Al}$

From [1],
$$\frac{\dfrac{0.01 + \Delta d_{Al}}{d_{Brass}}}{\dfrac{\Delta d_{Al}}{d_{Al}}} = \frac{\alpha_B}{\alpha_{Al}},$$

$$\text{or} \quad \left(\frac{10.01}{10.00}\right)\left(\frac{0.01 + \Delta d_{Al}}{\Delta d_{Al}}\right) = \frac{19}{24},$$

$$\text{or} \quad 0.01 + \Delta d_{Al} = \left(\frac{19}{24}\right)\left(\frac{10.01}{10.00}\right) \Delta d_{Al}, \quad [2]$$

$$\text{or} \quad 0.01 = -0.208\, \Delta d_{Al},$$

$$\Delta d_{Al} = -0.0482$$

$$\therefore \quad \frac{\Delta d_{Al}}{d_{Al}} = \alpha_{Al}\, \Delta T, \quad \Delta T = \frac{-0.0482}{(10.01)(24 \times 10^{-6})}$$

$$\therefore \quad \Delta T = -200.6$$

$$\therefore \quad T - 20 = -200.6, \quad \underline{T = -180.6\ ^\circ C}$$

(b) From [2] above,

$$0.02 + \Delta d_{Al} = \left(\frac{19}{24}\right)\left(\frac{10.02}{10.00}\right) \Delta d_{Al},$$

$$\text{or} \quad \Delta d_{Al} = -0.0967$$

$$\therefore \quad \Delta T = \frac{-0.0967}{(10.02)(24 \times 10^{-6})} = -402.3\ ^\circ C$$

$\therefore T-20 = -402.3, \; \underline{T = -382.3\,^\circ C},$

so <u>not possible</u> since below $-273\,^\circ c$

13. A pair of eyeglass frames is made of epoxy plastic. At room temperature (20.0°C), the frames have circular lens holes 2.20 cm in radius. To what temperature must the frames be heated if lenses 2.21 cm in radius are to be inserted in them? The average coefficient of linear expansion for epoxy is $1.30 \times 10^{-4}\ (^\circ C)^{-1}$.

$C = \pi d, \quad \Delta C = \pi \Delta d, \quad \therefore \dfrac{\Delta C}{C} = \dfrac{\pi \Delta d}{\pi d} = \dfrac{\Delta d}{d}$

$\therefore \dfrac{\Delta d}{d} = \alpha \Delta T, \quad \Delta d = \alpha \Delta T d$

$\therefore \Delta d = 0.01\,cm = (1.3 \times 10^{-4})(T-20)(2.20\,cm)$

$\therefore 35.0 = T-20, \quad \underline{T = 55^\circ C}$

16. The average coefficient of volume expansion for carbon tetrachloride is $5.81 \times 10^{-4}\ (^\circ C)^{-1}$. If a 50.0-gal steel container is filled completely with carbon tetrachloride when the temperature is 10.0°C, how much will spill over when the temperature rises to 30.0°C?

For steel, $\alpha = 11 \times 10^{-6}$, vol change occurs for steel, and $\beta_{steel} = 3\alpha = 33 \times 10^{-6} = 0.33 \times 10^{-4}$

$\therefore \Delta V_{steel} = \beta_{st} V_i \Delta T$

$$\Delta V_{CCl_4} = \beta_{CCl_4} V_i \Delta T$$

V_i is the same for both (50.0 gal)

$$\therefore \Delta V_{CCl_4} - \Delta V_{steel} = (5.81 - 0.33)(10^{-4})(50.0)(20.0\,°C)$$

$$= \underline{\underline{0.548 \text{ gal}}}$$

23.]A hollow aluminum cylinder 20.0 cm deep has an internal capacity of 2.000 L at 20.0°C. It is completely filled with turpentine and then slowly warmed to 80.0°C. (a) How much turpentine overflows? (b) If the cylinder is then cooled back to 20.0°C, how far below the cylinder's rim does the turpentine's surface recede?

(a) $\frac{\Delta V}{V_i} = \beta \Delta T$, and $\beta = 3\alpha$

$$\therefore \Delta V_{Turp} - \Delta V_{Al} = (\beta_{Turp} - \beta_{Al}) \Delta T (V_i)$$

$$= (\beta_{Turp} - 3\alpha_{Al}) \Delta T V_i$$

$$= [9 \times 10^{-4} - 3(24 \times 10^{-6})] 60\,°C (V_i)$$

$$= 8.28 \times 10^{-4} (60)(2000 \text{ cc})$$

$$= 99.4 \text{ cm}^3$$

(6) Assume base of cylinder is aluminum.

Turpentine, at $80°C$, was $(2000)(9\times10^{-4})(60) + 2000 = 108 + 2000 = 2108$ cc.

$\therefore \%\ \text{loss} = \frac{99.4}{2108} = 0.0472$

$\therefore$ Volume of Turpentine at $20°C$ will be 4.72% less. $\therefore$ Each linear dimension will be $(4.72\%)\frac{1}{3}$ less.

Consider $V - \rho V$, $\rho = \%$ loss of volume

$\therefore$ resultant volume is $V(1-\rho)$

Let $s = \%$ loss of each linear dimension.

$$\therefore V(1-\rho) = (l - sl)(w - sw)(h - sh)$$

$$= lwh(1-s)^3 = V(1-s)^3$$

$$= V(1 - 3s + 3s^2 - s^3)$$

For small s, $s^2 \ll 1$, $s^3 \ll 1$, so

$$V(1-\rho) \doteq V(1-3s), \text{ so } \rho = 3s,$$

$$s = \tfrac{1}{3}\rho$$

$\therefore (\frac{1}{3})(4.72\%)(20) = \underline{0.314 \text{ cm}}$

29.] The mass of a hot-air balloon and its cargo (not including the air inside) is 200 kg. The air outside is at 10.0°C and 101 kPa. The volume of the balloon is 400 m^3. To what temperature must the air in the balloon be heated before the balloon will lift off? (Air density at 10.0°C is 1.25 kg/m^3.)

Bouyant force = weight of displaced volume

$= (\rho_{air\ outside})(Vol)\, g$

Weight of air inside baloon is $\rho_{in}(Vol)g$

Assume balloon does not stretch.

$\therefore Vol = 400\ m^3$

$\therefore \rho_{out}(400)g - \rho_{in}(400)g - 200g = 0$, or

$2\rho_{out} - 2\rho_{in} - 1 = 0$, or $\rho_{out} - \rho_{in} = 0.5$

$\rho_{out} = 1.25\ Kg/m^3$, $\therefore$ want $\rho_{in} = 0.75\ Kg/m^3$

If balloon doesn't stretch, $P_{outside} = P_{inside}$

$\therefore PV = nRT$, or $\frac{P}{RT} = \frac{n}{V}$

But $\frac{n}{V} \propto$ density, and since Pressure is the same, $\rho \propto \frac{1}{T}$

$$\therefore \frac{\rho_{out}}{\rho_{in}} = \frac{T_{in}}{T_{out}}, \text{ so } T_{in} = T_{out}\left(\frac{\rho_{out}}{\rho_{in}}\right)$$

$$= 283K\left(\frac{1.25}{0.75}\right)$$

$$= 472K$$

$$= \underline{\underline{199\,^{\circ}C}}$$