Chapter 20 - Heat and the First Law of Thermodynamics

Note Title 1/31/2007

5. A 1.50-kg iron horseshoe initially at 600°C is dropped into a bucket containing 20.0 kg of water at 25.0°C. What is the final temperature? (Ignore the heat capacity of the container, and assume that a negligible amount of water boils away.)

$C_{Fe} = 448 \text{ J/kg-°C} \qquad C_{H_2O} = 4186 \text{ J/kg-°C}$

Let T = final temp.

$\therefore (20 \text{ kg})(4{,}186 \text{ J/kg-°C})(T-25) =$

$(1.50 \text{ kg})(448 \text{ J/kg-°C})(600-T)$

$83720(T-25) = 672(600-T)$

$124.58(T-25) = 600-T$

$125.58T = 600 + 3114.58$

$\underline{T = 29.6°C}$

11. A water heater is operated by solar power. If the solar collector has an area of 6.00 m^2 and the intensity delivered by sunlight is 550 W/m^2, how long does it take to increase the temperature of 1.00 m^3 of water from 20.0°C to 60.0°C?

$(6.00 \text{ m}^2)(550 \text{ W/m}^2) = 3300 \text{ J/sec}$

$1\,m^3\ H_2O = (10^2\,cm)^3 = 10^6\,cc\ H_2O = 10^6\,g$

$= 10^3\,Kg\ H_2O$

$\therefore \Delta T (3300\ J/sec) = (10^3\,kg)(4186\ J/kg\text{-}^\circ c)(40^\circ C)$

$\therefore \Delta T = \dfrac{(4.186 \times 10^6)(40)}{3.3 \times 10^3} = 5.07 \times 10^4\ sec$

$= \underline{14.1\ hr}$

17.] In an insulated vessel, 250 g of ice at 0°C is added to 600 g of water at 18.0°C. (a) What is the final temperature of the system? (b) How much ice remains when the system reaches equilibrium?

(a) Energy from melting ice is:

$(0.250\,Kg)(3.33 \times 10^5\ J/Kg) = 8.33 \times 10^4\ J$

To supply this energy, the 600g H_2O must drop

$(0.600\,Kg)(4186\ J/kg\text{-}^\circ C)\,\Delta T = 8.33 \times 10^4\ J,$

$\Delta T = 33.2^\circ C$

∴ Not enough energy to melt all of the ice.

∴ Dropping 18°C provides:

$$(0.600\ kg)(4186\ J/kg\text{-}°C)(18) = 4.52 \times 10^4\ J$$

This amount of energy will melt:

$$\frac{4.52 \times 10^4\ J}{3.33 \times 10^5\ J/kg} = 0.136\ Kg = 136\ g\ ice$$

∴ Final system, at 0°C, will be

(b) $250 - 136 = 114$ g ice floating in 736 g H_2O.

33. An ideal gas initially at 300 K undergoes an isobaric expansion at 2.50 kPa. If the volume increases from 1.00 m^3 to 3.00 m^3 and 12.5 kJ is transferred to the gas by heat, what are (a) the change in its internal energy and (b) its final temperature?

(a) During expansion, $P\Delta V = (2.50 \times 10^3)(3.00 - 1.00)$
$= 5.00 \times 10^3\ J$

It loses this energy by expanding.

$12.5 \times 10^3\ J$ is added

$\therefore\ 12.5 - 5.0 = \underline{7.5 \times 10^3\ J} = \Delta E_{int}$

(b) $P_1 V_1 = nRT_1$, $P_2 V_2 = nRT_2$, here $P_1 = P_2$

$\therefore\ \frac{V_1}{V_2} = \frac{T_1}{T_2}$, $T_2 = (300\,°K)\left(\frac{3.00\,m^2}{1.00\,m^2}\right)$

$= \underline{900°K}$

34. One mole of an ideal gas does 3 000 J of work on its surroundings as it expands isothermally to a final pressure of 1.00 atm and volume of 25.0 L. Determine (a) the initial volume and (b) the temperature of the gas.

For isothermal process, $W = nRT \ln\left(\frac{V_f}{V_i}\right)$

Here, $P_1 V_1 = nRT = P_2 V_2$. $\therefore\ W = P_f V_f \ln\left(\frac{V_f}{V_i}\right)$

(a) $\therefore\ \ln\left(\frac{V_f}{V_i}\right) = \frac{3000}{(1\,atm)(25\,L)}$ $\qquad 1\,atm = 1.013 \times 10^5\ N/m^2$

$\therefore\ \ln\left(\frac{25}{V_i}\right) = \frac{3000}{(1.01 \times 10^5)\,25}$

$1\,L = 1{,}000\ cm^3 = 1 \times 10^3 \times (10^{-6})\ m^3$
$= 1 \times 10^{-3}\ m^3$

$\therefore\ \ln\left(\frac{25 \times 10^{-3}}{V_i}\right) = \frac{3000}{(1.01 \times 10^5)(25 \times 10^{-3})} = 1.19$

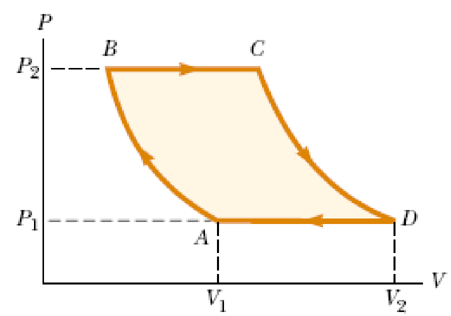
$$\therefore \quad \frac{25\times10^{-3}}{V_i} = 3.28, \quad V_i = \underline{7.62\times10^{-3}\ m^3}$$

$$= 7.62\ L$$

(b) $$T = \frac{PV}{nR} = \frac{(1\ atm)(25\ L)}{(1\ mol)\left(0.082\ \frac{L\cdot atm}{mol\cdot K}\right)} = \underline{305\ K}$$

39. An ideal gas is carried through a thermodynamic cycle consisting of two isobaric and two isothermal processes as shown in Figure P20.69. Show that the net work done on the gas in the entire cycle is given by

$$W_{\text{net}} = -P_1(V_2 - V_1)\ln\frac{P_2}{P_1}$$



$A \to B$: $W = nRT\ln\left(\frac{V_1}{V_B}\right)$ (on the gas)

Note: $P_1V_1 = nRT$, so $T = \frac{P_1V_1}{nR}$

Also, $P_2V_B = P_1V_1$, so $V_B = \frac{P_1V_1}{P_2}$

$$\therefore\ W = nR\left(\frac{P_1V_1}{nR}\right)\ln\left(\frac{V_1}{\frac{P_1V_1}{P_2}}\right)$$

$$= \underline{P_1V_1\ln\left(\frac{P_2}{P_1}\right)}$$

$B \to C$: $W = -P_2(V_C - V_B)$, negative since the gas expands.

from above, $V_B = \frac{P_1}{P_2} V_1$

Also, since $P_1 V_2 = P_2 V_C$, $V_C = \frac{P_1}{P_2} V_2$

$$\therefore W = -P_2\left(\frac{P_1}{P_2} V_2 - \frac{P_1}{P_2} V_1\right)$$

$$= \underline{-P_1(V_2 - V_1)}$$

$C \to D$: $W = -nRT \ln\left(\frac{V_2}{V_C}\right)$, negative since gas expands.

Since CD is an isotherm, $P_2 V_C = P_1 V_2$,
so $V_C = \frac{P_1}{P_2} V_2$

Also $P_1 V_2 = nRT$, so $T = \frac{P_1 V_2}{nR}$

$$\therefore W = -nR\left(\frac{P_1 V_2}{nR}\right) \ln\left(\frac{V_2}{\frac{P_1}{P_2} V_2}\right)$$

$$= \underline{-P_1 V_2 \ln\left(\frac{P_2}{P_1}\right)}$$

$D \to A: \quad W = \underline{P_1 (V_2 - V_1)}$

$\therefore$ Adding, $P_1 V_1 \ln\left(\frac{P_2}{P_1}\right) - P_1 (V_2 - V_1)$

$- P_1 V_2 \ln\left(\frac{P_2}{P_1}\right) + P_1 (V_2 - V_1)$

$= P_1 (V_1 - V_2) \ln\left(\frac{P_2}{P_1}\right)$

= work done <u>on</u> the gas for the cycle.

(which is < 0 since $V_2 > V_1$, $P_2 > P_1$).

40. In Figure P20.40, the change in internal energy of a gas that is taken from A to C is $+800$ J. The work done on the gas along path ABC is -500 J. (a) How much energy must be added to the system by heat as it goes from A through B to C? (b) If the pressure at point A is five times that of point C, what is the work done on the system in going from C to D? (c) What is the energy exchanged with the surroundings by heat as the cycle goes from C to A along the green path? (d) If the change in internal energy in going from point D to point A is $+500$ J, how much energy must be added to the system by heat as it goes from point C to point D?

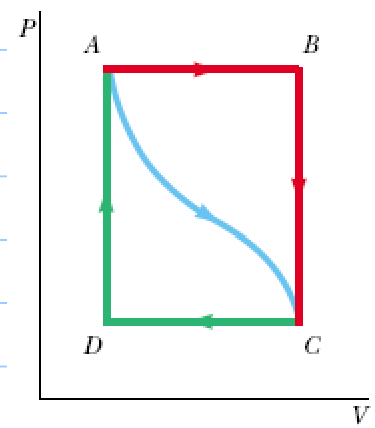


Figure P20.40

(a) $\Delta E_{int\, A \to C} = W_{A \to C} + Q_{A \to C}$

$= -500\text{ J} + Q = 800\text{ J}, \quad Q = \underline{1300\text{ J}}$

(b) $W_{C\to D}$ is isobaric, so $= P_C(V_C - V_D)$

$$= P_C(V_B - V_A) = \frac{1}{5}P_A(V_B - V_A)$$

But $W_{A\to B} = -500 = P_A(V_A - V_B)$

(negative since gas expands).

$$\therefore \frac{1}{5}P_A(V_B - V_A) = \underline{100\,J}$$

(c) $\Delta E_{int\, C\to A} = -800$

If 100 J is added to system by work by (b), and work $D\to A = 0$ (since $\Delta V = 0$), then

$$\Delta E_{int} = Q + W = Q + 100 = -800,$$

$\underline{Q = -900\,J}$

(d) $\Delta E_{int\, C\to D\to A} = -800\,J$

$$= \Delta E_{int\, C\to D} + \Delta E_{int\, D\to A}$$

$$= \Delta E_{int\, C\to D} + 500\,J$$

$$= Q_{C\to D} + W_{C\to D} + 500\,J$$

$$= Q_{C\to D} + 100\,J + 500\,J$$

$$\therefore Q_{CD} = -800 - 100 - 500 = \underline{-1400\,J}$$

43. A glass window pane has an area of 3.00 m^2 and a thickness of 0.600 cm. If the temperature difference between its faces is 25.0°C, what is the rate of energy transfer by conduction through the window?

$$P = kA\left|\frac{dT}{dx}\right| = \left(0.8\,\frac{W}{m^2\,{}^\circ C}\right)\left(3.00\,m^2\right)\left(\frac{25\,{}^\circ C}{0.006\,m}\right)$$

$$= \underline{10{,}000\,W}$$

44. A thermal window with an area of 6.00 m^2 is constructed of two layers of glass, each 4.00 mm thick, and separated from each other by an air space of 5.00 mm. If the inside surface is at 20.0°C and the outside is at −30.0°C, what is the rate of energy transfer by conduction through the window?

$$P = \frac{A\,\Delta T}{\sum_i L_i/k_i} = \frac{(6.00\,m^2)(50\,{}^\circ c)}{\frac{0.004}{0.8} + \frac{0.005}{0.023} + \frac{0.004}{0.8}}$$

$$= \frac{300}{0.227} = 1.32\times10^3\,W = \underline{1.32\,kW}$$