Chapter 22 - Heat Engines, Entropy, and the Second Law of Thermodynamics

Note Title 9/10/2007

11.] An ideal gas is taken through a Carnot cycle. The isothermal expansion occurs at 250°C, and the isothermal compression takes place at 50.0°C. The gas takes in 1 200 J of energy from the hot reservoir during the isothermal expansion. Find (a) the energy expelled to the cold reservoir in each cycle and (b) the net work done by the gas in each cycle.

(a) $e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h}$, so $\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$

$\therefore \frac{Q_c}{1200\text{ J}} = \frac{323\text{ K}}{523\text{ K}}$, $Q_c = \underline{741\text{ J}}$

(b) $1200 - 741 = \underline{259\text{ J}}$

19.]In a cylinder of an automobile engine, just after combustion, the gas is confined to a volume of 50.0 cm^3 and has an initial pressure of 3.00×10^6 Pa. The piston moves outward to a final volume of 300 cm^3, and the gas expands without energy loss by heat. (a) If $\gamma = 1.40$ for the gas, what is the final pressure? (b) How much work is done by the gas in expanding?

(a) $P_1V_1^\gamma = P_2V_2^\gamma$, so

$(3.00 \times 10^6\text{ Pa})(50.0\text{ cm}^3)^{1.40} = P_2(300\text{ cm}^3)^{1.40}$

$P_2 = \underline{2.44 \times 10^5\text{ Pa}}$

(b) Use $\int_{V_1}^{V_2} P\,dV$. Since $PV^\gamma = K$, $P = \frac{K}{V^\gamma}$

and $K = P_1 V_1^\gamma$ from (a)

$$\therefore W = \int_{V_1}^{V_2} \frac{K\,dV}{V^\gamma} = \frac{K V^{1-\gamma}}{1-\gamma}\Bigg|_{V_1}^{V_2} = \frac{K V^{1-\gamma}}{\gamma-1}\Bigg|_{V_2}^{V_1}$$

$$K = P_1 V_1^\gamma = (3.00\times10^6 Pa)(50.0 cm^3)^{1.4} = 7.17\times10^8$$

$$\gamma - 1 = 0.4 \quad \therefore \frac{K}{\gamma-1} = 1.79\times10^9$$

$$V^{1-\gamma}\Big|_{V_2}^{V_1} = \frac{1}{V^{\gamma-1}}\Bigg|_{V_2}^{V_1} = \frac{1}{(50)^{0.4}} - \frac{1}{(300)^{0.4}}$$

$$= 0.107$$

$$\therefore W = (1.79\times10^9)(0.107)(10^{-2} m/cm)^3$$

$$= \underline{\underline{192\ J}}$$