# Chapter 23 - Electric Fields

Note Title 11/7/2007

1. (a) Calculate the number of electrons in a small, electrically neutral silver pin that has a mass of 10.0 g. Silver has 47 electrons per atom, and its molar mass is 107.87 g/mol. (b) Electrons are added to the pin until the net negative charge is 1.00 mC. How many electrons are added for every $10^9$ electrons already present?

(a) $10\,g = \left(\frac{10\,g}{107.87\,g/mole}\right)\left(6.02\times10^{23}\,\frac{atoms}{mole}\right)\frac{47\,elec.}{atom}$

$= \underline{\underline{2.62\times10^{24}}}$ electrons

(b) $1.00\,mC = 10^{-3}\,C$, so $(10^{-3}\,C)\left(6.24\times10^{18}\,\frac{elec}{C}\right)$

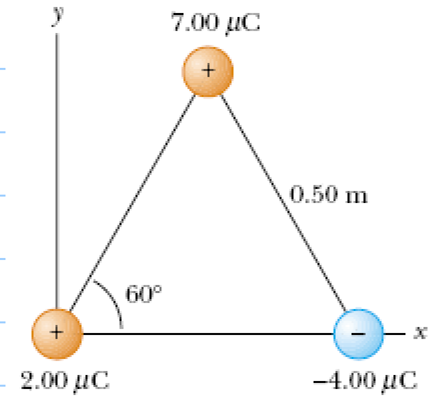
$= 6.24\times10^{15}$ electrons

There are $\frac{2.62\times10^{24}}{10^9} = 2.62\times10^{15}$ packets of $10^9$ electrons.

$\therefore \frac{6.24\times10^{15}}{2.62\times10^{15}} = \underline{2.38}$

$\therefore$ 2.38 elec. added for every $10^9$ elec. already present.

7. Three point charges are located at the corners of an equilateral triangle as shown in Figure P23.7. Calculate the resultant electric force on the 7.00-$\mu$C charge.




From the 2.00 $\mu C$ charge,

$$F_2 = \frac{(8.99\times10^9)(2\times10^{-6}C)(7\times10^{-6}C)}{(0.5m)^2}$$

$$= 503\times10^{-3}\,N \text{ at } 60^\circ \text{ up from horiz.}$$

$$= 0.251\,N\,\hat{i} + 0.436\,N\,\hat{j}$$

From the $-4.00\mu C$ charge,

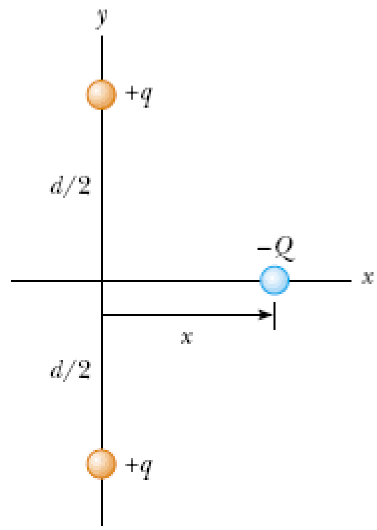
$$F_4 = \frac{(8.99\times10^9)(7\times10^{-6}C)(4\times10^{-6}C)}{(0.5m)^2}$$

$$= 1.01\,N \text{ at } 60^\circ \text{ down from horiz.}$$

$$= 0.503\,N\,\hat{i} - 0.872\,N\,\hat{j}$$

$$\therefore F_2 + F_4 = 0.754\,N\,\hat{i} - 0.436\,N\,\hat{j}$$

10. **Review problem.** Two identical particles, each having charge $+q$, are fixed in space and separated by a distance $d$. A third point charge $-Q$ is free to move and lies initially at rest on the perpendicular bisector of the two fixed charges a distance $x$ from the midpoint between the two fixed charges (Fig. P23.12). (a) Show that if $x$ is small compared with $d$, the motion of $-Q$ will be simple harmonic along the perpendicular bisector. Determine the period of that motion. (b) How fast will the charge $-Q$ be moving when it is at the midpoint between the two fixed charges, if initially it is released at a distance $a \ll d$ from the midpoint?




a. Need to show force on $-Q$ is a linear restoring force function of $x$, if $x \ll d$.

Vertical forces on $-Q$ from each $+q$ cancel.
Horizontal forces are identical for each $+q$.

$$F_h = \frac{k_e q(-Q)}{x^2 + \frac{d^2}{4}} \cos\theta, \quad \theta = \text{angle } \overline{qQ} \text{ to horizontal.}$$

$$\cos\theta = \frac{x}{\sqrt{x^2 + \frac{d^2}{4}}}$$

$$\therefore \text{Total force} = 2F_h = \frac{-2k_e qQx}{\left(x^2 + \frac{d^2}{4}\right)^{3/2}}$$

If $x << d$, then $x^2 << \frac{d^2}{4}$, so

$$F = -\frac{2k_e q Q x}{\left(\frac{d^2}{4}\right)^{3/2}} = -\frac{16 k_e q Q x}{d^3} = -Kx$$

$\therefore$ a linear restoring force of $x$, so simple harmonic motion.

For period, $K = \frac{16 k_e q Q}{d^3}$, $T = 2\pi\sqrt{\frac{m}{K}}$

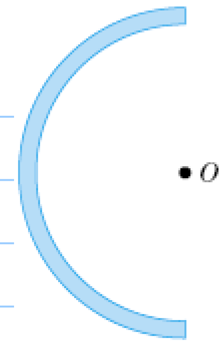
$$\therefore \underline{T} = 2\pi\sqrt{\frac{m d^3}{16 k_e q Q}} = \underline{\frac{\pi}{2}\sqrt{\frac{m d^3}{k_e q Q}}}$$

(b) $V_{max} = \omega(\text{amplitude}) = \omega a$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{\pi}{2}\sqrt{\frac{md^3}{k_e q Q}}} = \frac{4}{\sqrt{\frac{md^3}{k_e q Q}}}$$

$$\therefore V_{max} = \omega a = \frac{4a}{\sqrt{\frac{md^3}{k_e q Q}}} = 4a\sqrt{\frac{k_e q Q}{md^3}}$$

33. ] A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown in Figure P23.33. The rod has a total charge of $-7.50\ \mu C$. Find the magnitude and direction of the electric field at $O$, the center of the semicircle.




$\pi r = 14.0$ cm, $r = \frac{14}{\pi}$ cm

Charges on opposite sides cancel in the y-direction.


$\therefore$ For any point, $E_e \sin\theta$ is the contribution in the x-direction.

$\therefore$ Integrate from $\theta = 0°$ to $180°$, or $\theta = 0$ to $\pi$ radians.

Let $\lambda$ = charge density = $\frac{Q}{L}$

$r\,d\theta$ = segment of semicircle.

$\therefore$ charge of segment = $\lambda r\, d\theta$

$$\therefore dE = \frac{k_e \lambda r\, d\theta \sin\theta}{r^2} = \frac{k_e \lambda \sin\theta\, d\theta}{r}$$

$$\therefore E = \frac{k_e \lambda}{r} \int_0^{\pi} \sin\theta\, d\theta = \frac{k_e \lambda}{r} \left(-\cos\theta\right)\Big|_0^{\pi}$$

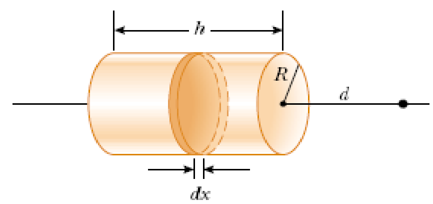
$$= \frac{k_e \lambda}{r}\left[1+1\right] = \frac{2k_e\lambda}{r}$$

$$\lambda = \frac{-7.50\,\mu C}{L} \quad \left[r = \frac{L}{\pi},\ L = 14.0\,cm\right]$$

$$\therefore E = \frac{-2(8.99\times10^{9})(-7.50\times10^{-6})\pi^2}{(0.14)^2}$$

$$= -2.16\times10^{7}\ \text{Newtons/Coul.}$$

34. (a) Consider a uniformly charged thin-walled right circular cylindrical shell having total charge $Q$, radius $R$, and height $h$. Determine the electric field at a point a distance $d$ from the right side of the cylinder as shown in Figure P23.34. (*Suggestion:* Use the result of Example 23.8 and treat the cylinder as a collection of ring charges.) (b) **What If?** Consider now a solid cylinder with the same dimensions and carrying the same charge, uniformly distributed through its volume. Use the result of Example 23.9 to find the field it creates at the same point.




(a) From Example 23.8, $dE_x = \dfrac{k_e(d+x)}{((d+x)^2+R^2)^{3/2}}\,dQ$

Here, $dQ = \dfrac{dx}{h}Q$

$$\therefore \int E_x = \int_0^h \frac{k_e(d+x)}{\left[(d+x)^2+R^2\right]^{3/2}}\frac{Q}{h}\,dx$$

Letting $y = d + x$,

$$\int E_x = \frac{K_e Q}{h} \int_d^{d+h} \frac{y}{(y^2 + R^2)^{3/2}} dy$$

$$= K_e \frac{Q}{h} \left[ - \frac{1}{\sqrt{y^2 + R^2}} \right]_d^{d+h}$$

$$\therefore E = K_e \frac{Q}{h} \left[ \frac{1}{\sqrt{d^2 + R^2}} - \frac{1}{\sqrt{(d+h)^2 + R^2}} \right],$$

pointed to right along axis of $d$.

(b) From example 23.9,

$$dE_x = 2\pi K_e \sigma_{dx} \left( \frac{d+x}{|d+x|} - \frac{(d+x)}{((d+x)^2 + R^2)^{1/2}} \right)$$

$$\text{Charge/volume} = \frac{Q}{\pi R^2 h}$$

$\therefore$ charge for $dx$ disk is: $\left( \frac{Q}{\pi R^2 h} \right) \left( \pi R^2 dx \right) = \frac{Q}{h} dx$

$\therefore$ surface charge for $dx$ disk is: $\frac{Q}{\pi R^2 h} dx$

$$\therefore dE_x = \frac{2K_e Q}{R^2 h}\left(\frac{d+x}{|d+x|} - \frac{(d+x)}{((d+x)^2+R^2)^{1/2}}\right)dx$$

Let $y = d+x$, and here, $d+x$ is always $>0$ since $x$ roams from $0 \to h$.

$$\therefore dE_x = \frac{2K_e Q}{R^2 h}\left(1 - \frac{y}{(y^2+R^2)^{\frac{1}{2}}}\right)dy$$

$$\therefore E = \int dE_x = \frac{2K_e Q}{R^2 h}\int_d^{d+h}\left(1 - \frac{y}{\sqrt{y^2+R^2}}\right)dy$$

$$= \frac{2K_e Q}{R^2 h}\left[y - \sqrt{y^2+R^2}\right]_d^{d+h}$$

$$= \frac{2K_e Q}{R^2 h}\left[h + \sqrt{d^2+R^2} - \sqrt{(d+h)^2+R^2}\right]$$

pointed to right along $d$ axis.