2.2 The Idea of Elimination

Note Title 9/27/2006

4. What multiple l of equation 1 should be subtracted from equation 2?

$$ax + by = f$$
$$cx + dy = g$$

The first pivot is "a" (assumed non-zero). Elimination produces what formula for the second pivot? What is y? The second pivot is missing when $ad = bc$.

(a) $l = \frac{c}{a}$: $\quad cx + \frac{bc}{a} y = \frac{fc}{a} \quad (1')$

$$\therefore \quad 0x + \left(d - \frac{bc}{a}\right) y = g - \frac{fc}{a} \quad (2')$$

$\therefore$ 2nd pivot: $d - \frac{bc}{a}$

(b) $$y = \frac{g - \frac{fc}{a}}{d - \frac{bc}{a}} = \frac{ag - fc}{ad - bc}$$

5. Choose a right side which gives no solution and another right side which gives

infinitely many solutions. What are two of those solutions?

$$3x + 2y = 10$$
$$6x + 4y = z$$

(a) No solution : $z \neq 20$

Infinitely many solutions : $z = 20$

From (4), here $ad = bc$, so $0y = \frac{ag - fc}{a}$

$\therefore$ if $ag - fc = 0$, infinitely many solutions

if $ag - fc \neq 0$, no solution.

$\therefore$ look at $3z - 60$. $z = 20$ is critical value.

6. Choose a coefficient "b" that makes this system singular. Then choose a right side g that makes it solvable. Find two solutions in that singular case.

$$2x + by = 16$$
$$4x + 8y = g$$

(a) Singular $\Rightarrow$ no second pivot, so, from (4),

$(2)(8) - 4b = 0$, $\therefore$ $\underline{b = 4}$

(b) From (4), solvable (i.e., infinitely many solutions) if $2g-(4)(16)=0$, $g = \underline{32}$

(c) $(0,4)$ and $(8,0)$ are solutions when $g = 32$.

25. For which two numbers "a" will elimination fail

on $A = \begin{bmatrix} a & 2 \\ a & a \end{bmatrix}$?

$a^2 - 2a = 0 \Rightarrow a = 0, 2$

26. For which three numbers "a" will elimination fail to give three pivots?

$$A = \begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix}$$

$$\begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix} \rightarrow \begin{bmatrix} a & 2 & 3 \\ 0 & a-2 & 1 \\ 0 & a-2 & a-3 \end{bmatrix} \rightarrow \begin{bmatrix} a & 2 & 3 \\ 0 & a-2 & 1 \\ 0 & 0 & a-4 \end{bmatrix}$$

$\therefore a = 0, 2, 4$