2.3 Elimination Using Matrices

Note Title 10/4/2006

1. Write down the 3x3 matrices that produce these elimination steps:

(a) E_{21} subtracts 5 times row 1 from row 2.

$$\begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) E_{32} subtracts -7 times row 2 from row 3.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 7 & 1 \end{bmatrix}$$

(c) P exchanges rows 1 and 2, then rows 2 and 3

$$P = P_{23} \cdot P_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

2. In problem 1, applying E_{21} and then E_{32} to the column $b = (1,0,0)$ gives $E_{32}E_{21}b =$

$\underline{(1, -l_{21}, l_{32}l_{21})}$. Applying E_{32} before E_{21} gives $E_{21}E_{32}b = \underline{(1, -l_{21}, 0)}$

3. (a) What three matrices E_{21}, E_{31}, E_{32} put A into triangular form U?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \text{ and } E_{32}E_{31}E_{21}A = U$$

$$\underline{E_{21}} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \underline{E_{31}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\therefore E_{31}E_{21}A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 4 & 0 \end{bmatrix}$$

$$\therefore \underline{E_{32}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(b) Multiply those E's to get one matrix M that does the elimination: $MA = U$.

$$\underline{M} = E_{32} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{bmatrix}$$

5. Suppose $a_{33} = 7$ and the third pivot is 5. If you change a_{33} to 11, the third pivot is ___. If you change a_{33} to ___, there is no third pivot.

(a) Realize from the 2nd pivot operation, $-l_{31}$ is the multiplier, so

$$E_{21} A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$E_{31} E_{21} A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & a_{32}' & (-l_{31}) a_{13} + a_{33} \end{bmatrix}$$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -l_{32} & 1 \end{bmatrix},$$ so the third pivot

is $a_{33}' = (-l_{31}) a_{13} + a_{33} + (-l_{32}) a_{23}'$

$\therefore$ changing a_{33} from 7 to 11 means

a_{33}' changes from 5 to $5 + (11-7) = \underline{9}$

(b) To make a_{33}' change from 5 to 0, subtract 5 from a_{33}, or $7-5 = \underline{\underline{2}}$

9. (a) E_{21} subtracts row 1 from row 2 and then P_{23} exchanges rows 2 and 3. What matrix $M = P_{23}E_{21}$ does both steps at once?

Let $E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -l_{21} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $P_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$\therefore \; P_{23}E_{21} = \underline{\underline{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -l_{21} & 1 & 0 \end{bmatrix}}}$$

(b) P_{23} exchanges rows 2 and 3 and then E_{31} subtracts row 1 from row 3. What matrix $M = E_{31}P_{23}$ does both steps at once? Explain why the M's are the same but the E's are different.

$E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -l_{31} & 0 & 1 \end{bmatrix}$, $P_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$\therefore \; E_{31}P_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -l_{31} & 1 & 0 \end{bmatrix}$$

Here $-l_{31}$ in (b) $= -l_{21}$ in (a) because

rows 2 and 3 have already been exchanged, so $-l_{31} = -a_{21}/a_{11}$ which is $-l_{21}$ in (a).

28. If $AB = I$ and $BC = I$, use the associative law to prove $A = C$.

Pf: $AB = I$, so $(AB)C = IC = C$

$\therefore C = (AB)C = A(BC) = A(I) = A$