3.1 Spaces of Vectors

Note Title 3/23/2007

20. For which right sides (find a condition on b_1, b_2, b_3) are these systems solvable?

(a) $\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

(a) Each column is a scalar multiple of the other, so (b_1, b_2, b_3) must be of the form $(x, 2x, -x)$.

(b) $b_1 = -b_3$

22. For which vectors (b_1, b_2, b_3) do these systems have a solution?

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\text{and} \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is equivalent to I, so <u>any</u> (b_1, b_2, b_3) will work.

(b) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ last 2 cols are identical, so column space of matrix is that of $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

which is the same as $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\therefore$ if $\underline{b_3 = 0}$, b_1 and b_2 can be anything.

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ First 2 cols the same, so column space determined by $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\therefore$ Last 2 components of (x_1, x_2, x_3) are equal, so $b_2 = b_3$.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x_3$$

$$\Rightarrow \begin{aligned} x_1 + x_2 + x_3 &= b_1 \\ x_3 &= b_2 \\ x_3 &= b_3 \end{aligned}$$

$\therefore$ If $\underline{b_2 = b_3} = x_3$, b_1 can be anything

28. Construct a 3 by 3 matrix whose column space contains (1, 1, 0) and (1, 0, 1) but not (1, 1, 1). Construct a 3 by 3 matrix whose column space is only a line.

(a) You can't go $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ from $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ by a linear combination. $\therefore \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

(b) Choose column space of $\vec{o}$ and $\vec{r}$

$$\therefore \begin{bmatrix} r_1 & cr_1 & dr_1 \\ r_2 & cr_2 & dr_2 \\ r_3 & cr_3 & dr_3 \end{bmatrix}$$