7.3 Change of Basis

Note Title 10/7/2008

6. Suppose we have two bases $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$ and $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_n$ for $\mathbf{R}^n$. If a vector has coefficients b_i in one basis and c_i in the other basis, what is the change of basis matrix in $\boldsymbol{b} = M\boldsymbol{c}$? Start from

$$b_1\boldsymbol{v}_1 + \cdots + b_n\boldsymbol{v}_n = V\boldsymbol{b} = c_1\boldsymbol{w}_1 + \cdots + c_n\boldsymbol{w}_n = W\boldsymbol{c}.$$

Your answer represents $T(\boldsymbol{v}) = \boldsymbol{v}$ with input basis of $\boldsymbol{v}$'s and output basis of $\boldsymbol{w}$'s. Because of different bases, the matrix is not I.

Since $Vb = Wc$, $b = V^{-1}Wc$,
so $M = V^{-1}W$.

7. The dual basis vectors $\boldsymbol{w}_1^*, \ldots, \boldsymbol{w}_n^*$ are the columns of $W^* = (W^{-1})^{\mathrm{T}}$. Show that the original basis $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_n$ is "the dual of the dual." In other words, show that the $\boldsymbol{w}$'s are the rows of $(W^*)^{-1}$. **Hint:** Transpose the equation $WW^{-1} = I$.

The "dual of the dual" consists of the rows of $(W^*)^{-1}$

$$(W^*)^{-1} = \left[(W^{-1})^T\right]^{-1} = \left[(W^{-1})^{-1}\right]^T = W^T$$

$\therefore$ The rows of $(W^*)^{-1}$ are the rows of W^T, which are the columns of W, which are w_i.

This all derives from $(A^T)^{-1} = (A^{-1})^T$.